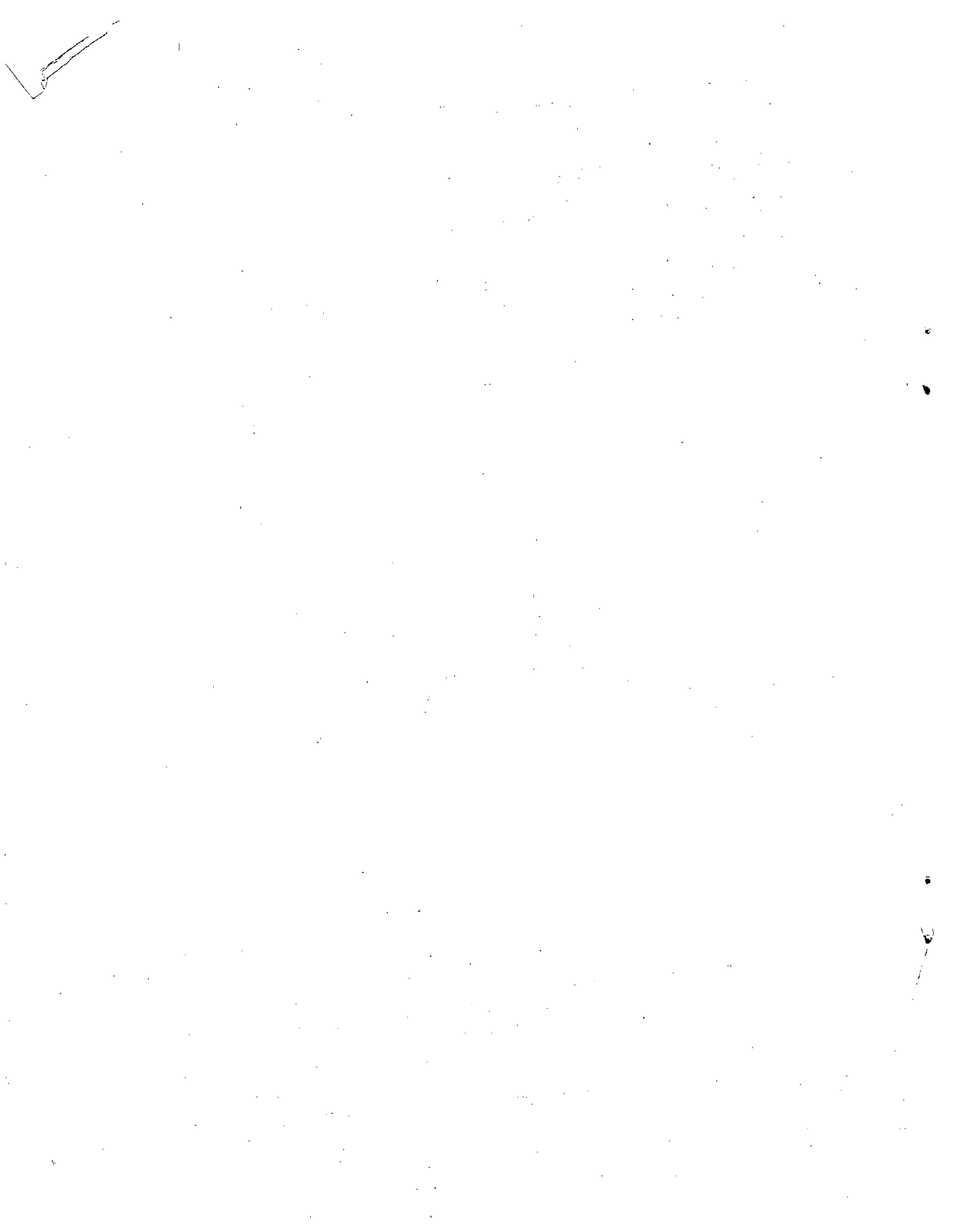


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The project was supervised by Dr. R. S. Ratner, Manager, Transportation Engineering and Control Group at Stanford Research Institute, Menlo Park, California.

I BACKGROUND AND PERSPECTIVE

The Federal Aviation Administration's Systems Research and Development Services and National Aviation Facilities Experimental Center (NAFEC) are engaged in developing and applying computer-aided systematic methodologies to the national network of (1) airways and (2) ground station VORs and VORTACs used to create airways and provide other services. Stanford Research Institute was awarded Contract No. DOT-FA71WA-2547 on 19 February 1971, to support the FAA in this effort. The work under the contract was to be done in close cooperation with a NAFEC team.

The primary objective of the contracted work was the development of the following two analytical tools:

- A systematic methodology and associated computer program(s) for the design of a national network of airway routes for the optimal movement of high-altitude (above 18,000 ft.) area navigation traffic flow. This is referred to as Tasks 2 and 3.
- A methodology and associated computer programs for the synthesis of a VOR/VORTAC network to provide navigational aids for the related airway routes. This is referred to as Task 4.

Task 1 is concerned with preparing suitable traffic demand models on the basis of present and forecast traffic data. These models are to be used as computer input data for development and testing of the Hi-RNAV networks. A discussion of Task 1 has been included as Appendix G of this part of the report, the rest of which is concerned with SRI's accomplishment to date in connection with Tasks 2 and 3.

Various models and techniques to design a national VORTAC grid are described in Part 2 of the report.

The question of combining the route network design and the VORTAC grid design is discussed in Sections IV and V, and Appendix I of this volume. A similar discussion is included in Part 2 of the report for the sake of making both parts of the report self-contained.

II SUMMARY

A. General

This draft report describes and explains the research done by SRI during the period from the awarding of the contract until the end of August 1972 to develop a computer-aided air route network design methodology. The methodology developed so far does not yield a network design directly. Rather, it consists mainly of tools--in the form of mathematical models and techniques--to assess various network attributes and to implement various design alternatives. A complete methodology giving a direct route network design requires acceptable, realistic network evaluation criteria based on various network attributes and expressed in concrete analytical terms. It is hoped that the results obtained so far and presented in this report will provide a basis for discussions and consultations with various experts in the field of aviation, after which it is expected that a complete design methodology can be accomplished.

B. Air Route Network Design Tools Developed to Date

The various tools and techniques that have been developed so far are briefly described below. These tools and techniques can eventually be incorporated in a complete design methodology.

1. Establishment of Route Intersection Points and Angles

Mathematical relationships and associated computer programs have been developed by which the intersection points of various routes or route sections and the intersection angles can be established. The

latitude and longitude of the start and end point of each route or route section must be specified. The mathematical relationships are based on assuming the earth to be a sphere. The transformation from geocentric to geodetic coordinates can easily be accomplished through standard transformation relationships.

2. Establishment of Various Network Attributes

Mathematical models and relationships and the associated computer programs have been developed by which several network attributes can be established, e.g.,

- Expected number of potential conflicts per unit time at various intersections along each route.
- Expected duration of the potential conflicts at various intersections.
- Capacity at various intersections, based on a specified number of acceptable potential conflicts and on conflict duration.
- Maximum value of the flow product at crossing routes, with an indication of the altitude level at which this value occurs.
- An indication if the existing flow product at any intersection exceeds a prespecified safety margin.
- Flight mileage.
- Route mileage.
- Intersection density count.
- Existing flows along various routes and route sections, and an indication if any flows exceed a prespecified limit.

The flow rate of aircraft at various altitudes between various communities must be specified. Note that safety and delay in an air traffic control (ATC) system are implicitly accounted for in the potential conflict count and the conflict duration, since a system producing fewer potential

conflicts or shorter potential conflict durations is expected to be safer and to result in less delay.

3. Merging and Bending Routes

Techniques and associated computer programs have been developed to merge closely located routes on the basis of ATC convenience and practical considerations. The possibility of bending a route at suitable places to bypass congested or restricted areas has been considered. A convenient systematic technique to implement this possibility has not yet been developed.

4. Computer and Plot Programs

A computer program has been developed in Fortran IV language to implement various steps of the design methodology and is described in detail in the User's Manual, "Computer Aided Traffic/Airway/VOR(TAC) Network Program," by John J. Penick and Kathryn N. Sapios dated October 1972. A program has also been developed to plot the air routes based on the output of the computer program. A description of the plot program is included in the User's Manual. The program is compatible with the IBM 7090, Sigma 5, and IBM System 360.

III PROJECT SCOPE, GROUND RULES, AND PROBLEM FORMULATION

A. Project Scope and Ground Rules

During the early stages of the project, the NAFEC and SRI team members held several discussions to establish the project scope and ground rules so as to limit the required efforts to manageable bounds. The scope and the various ground rules relating to Tasks 2 and 3 that were tentatively agreed upon between NAFEC and SRI are summarized below. For further details, refer to Appendix A.

- Traffic Data Base

The IFR peak day traffic tapes will be used as the data base for traffic demands.

- Interface with Terminal Areas

Departure and arrival points will be assumed to be located 50 nmi from the center of the community. Climb and descent will be regarded as terminal problems and will not be handled explicitly in the design of the route network.

- Restricted Airspace

Restricted airspace will be considered in the design of the route network.

- Route Widths

Route widths will be assumed to be constant for the design of the route network, at least during the first phase of the project.

- Altitude Utilization Rules

The hemispherical rule will be used as a starting point from which other altitude utilization rules will be investigated for possible inclusion in the Task 3 methodology.

- Atmospheric Conditions

Atmospheric conditions, i.e., weather, wind, and also jet stream, will not be considered explicitly in the initial design of the route network.

- Form of RNAV

All aircraft will be considered to be equipped with the $\rho - \theta$ type of RNAV equipment with slant-range correction.

- VORTAC Coverage

For the purpose of route network design, it will be assumed that full VORTAC coverage is available throughout the United States.

- High-Altitude Level

Only air route altitudes at or above 18,000 ft will be considered.

B. Identification of Design Variables

A route is defined by the horizontal projection that gives the geographic location of various points and by its altitude. Thus the design of a route network requires the design of the horizontal profile and the altitude for various routes. The horizontal profile is generated by bending or merging the route with another route or diverting it from some route; in some cases, the direct great-circle path between two communities may prove to be the desired horizontal profile. Furthermore, the use of RNAV equipment makes it possible to utilize parallel routes. Thus the three variables to be considered for the design of a route network are:

- Route bending (merging, diverging) in a horizontal plane
- Use of parallel routes
- Assignment of altitude.

C. An Initial Problem Formulation

In the past, air routes have been designed on an as-needed basis. The present effort may be the first to treat air route design as a system design problem. The formulation of the problem as a system design problem has required some simplifying assumptions, since otherwise the problem quickly becomes intractable.

After considering several factors that affect route network design, the NAFEC and SRI team members decided to formulate the problem tentatively in the following way. Given the traffic demand between various community pairs in terms of expected aircraft flows per unit time, design a route network (i.e., select the horizontal profile, altitude, and number of parallel routes) between respective community pairs such that:

- ATC constraints are fulfilled.
- A suitable combination of the following factors is accomplished
 - Expected number of potential conflicts
 - Expected conflict duration
 - Capacity of air routes
 - Capacity of intersections
 - Additional route length
 - Flight mileage
 - Number and density of route intersection points
 - Assignment of an altitude not necessarily that desired by the user.

It is expected that factors omitted or inadequately treated in the initial formulation can be introduced later, when a concrete methodology for the tentative formulation has been developed.

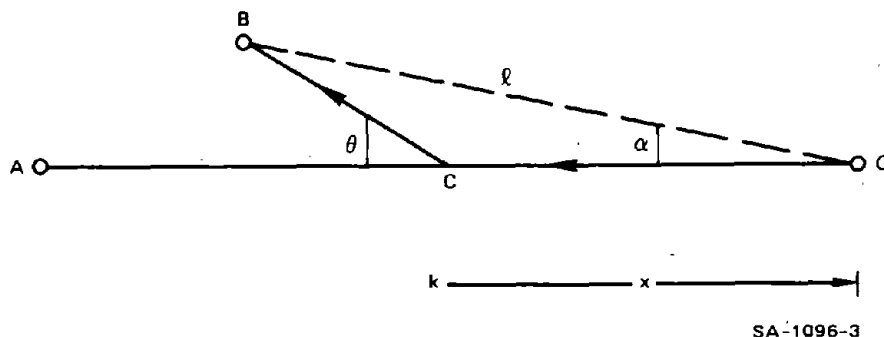
The above problem statement is essentially qualitative in nature. It was necessary to develop analytical models for all the features indicated so that a concrete and quantitative basis for comparing various route networks could be established. These analytical models are discussed in Section IV. A preliminary methodology to change various design variables and test the resulting network attributes is presented in Section V.

IV THE FAMILY OF NETWORK ATTRIBUTE MODELS

A. ATC Constraints

From the point of view of users flying between Communities i and j , the best route is the direct great-circle path between i and j , since this is the shortest route. However, there frequently are great-circle routes from Community i to Communities $j, k, l, m \dots$ such that the angles between Routes ij and ik , or ij and il , and so forth, are very small. For example, the direct great-circle route from New York to Denver has an initial heading of 278.13° and the route from New York to San Francisco has an initial heading of 281.61° . The difference between these two headings is only 3.48° . Under such circumstances, ATC controllers cannot treat these routes as two distinct airways for a considerable length, since the lateral separation minimum is not fulfilled. Such routes can be merged for a suitable distance to a point where the turn-off angle to the destination is sufficiently large that both the pilot and the ATC controller can easily accomplish it within a reasonably short time. An example of the merging of two routes is shown in Figure 1, using plane trigonometric relationships. A detailed discussion and application of spherical trigonometric relationships is included in Appendix B. When the value of θ (see Figure 1) is specified, e.g., 10° or 15° , it is easy to calculate the length of the merged part, $\overline{OC} = x$, in terms of the original length, $\ell = \overline{OB}$, and the initial angle, α . As shown in Appendix B, the length, x , and the increment in route length, $\Delta = (\overline{OC} + \overline{CB} - \overline{OB})$, are given by

$$x = \ell (\cos \alpha - \sin \alpha \cot \theta) \quad (1)$$



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FIGURE 1 MERGING OF A ROUTE ALONG ANOTHER ROUTE

$$\Delta = l \frac{\cos \left(\frac{\theta}{2} - \alpha \right)}{\cos \frac{\theta}{2}} - l \quad (2)$$

Appendix B also shows that, for a departing angle θ of 10° , the increase in the length of the route being merged does not exceed 0.4 percent of the direct route length and that, for a departing angle of 15° , the excess length is less than 0.9 percent.

B. Expected Number and Duration of Potential Conflicts

One of the most significant attributes of air route network and ATC system is safety. Unfortunately the expression safety is not easy to quantify. However, it is agreed that a flight is likely to be safer than another flight if it encounters fewer potential conflicts or if the potential conflicts are of shorter duration. Extending this approach to the entire network, it seems reasonable to state that a network which, for a given traffic flow, produces fewer expected potential conflicts or shorter total potential conflict duration is safer than a network producing more--at least, in an average sense. Furthermore, every potential conflict requires control action from ATC controllers. Thus ATC workload

is significantly dependent on the number and type of the potential conflicts to be resolved. Typically, ATC controller action consists of diverting one of two potentially conflicting aircraft, which means the introduction of delay. It therefore appears that safety, ATC workload, and delay are all heavily dependent upon potential conflicts and their duration. Consequently, an analytical model giving the expected number of potential conflicts and their duration would be a very useful tool in evaluating a network.

Mathematical models concerned with potential conflicts at an intersection as a function of intersection angle, traffic flow, separation minimum, and average aircraft speed have been developed and are discussed in Appendix C. A summary of these models is given below for convenience. All the models refer to the two intersecting routes, AB and CD, shown in Figure 2.

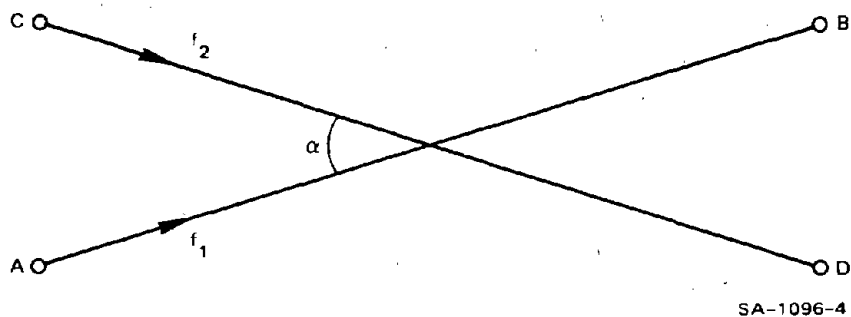


FIGURE 2 TWO INTERSECTING ROUTES SHOWING FLOWS OF AIRCRAFT PER HOUR (f_1, f_2)

1. Expected Number of Potential Conflicts per Hour

Let the radar separation minimum be N miles and the average speed of aircraft along Routes AB and CD be V mph. Then the expected number of potential conflicts per hour, denoted by E_c , is given by

$$E_c = 2 \frac{f_1 f_2}{V} \times \sec \frac{\alpha}{2} \text{ per hour} , \quad (3)$$

where f_1 and f_2 are the flow rates of aircraft per hour along AB and CD.

2. Expected Conflict Duration Time per Conflict

The expected duration of a potential conflict, τ , at an intersection is given by

$$\tau = \frac{\pi}{4} \frac{X}{V} \operatorname{cosec} \frac{\alpha}{2} \text{ hours} . \quad (4)$$

Note that τ does not depend on f_1 or f_2 . It indicates the expected duration of potential conflict at the intersection, which is a function of only α , X , and V .

3. Conflict Duration Time per Hour at the Intersection

The total time per hour, H , during which various aircraft will remain in conflict if nothing is done by ATC, is given by

$$H = \pi \frac{X^2}{2V} f_1 f_2 \operatorname{cosec} \alpha . \quad (5)$$

The usefulness of this number lies in indicating the type of conflict and the relative effort required by the controller to resolve it.

C. Capacity of an Air Route

1. Capacity Based on Average Speed and a Safety Factor

Let V be the average speed of an aircraft flying along a route, and let y be the longitudinal separation minimum. Then the capacity of the air route can be defined as

$$C_k = \frac{V}{(1 + k)y} \quad , \quad (6)$$

where k is a positive constant greater than or equal to zero. This constant is usually referred to as a safety factor. Thus, for $V = 600$ and $y = 20$, the capacity of an air route with zero safety factor would be 30 aircraft per hour. The capacity with a safety factor k of 5 would be six aircraft per hour, and so forth.

2. Capacity Based on a Certain Allowable Number of Overtakings

Let a single route be used by various types of aircraft having speeds $V_1, V_2, V_3 \dots$ mph, V_i being greater than V_{i+1} . Let f_i be the flow per hour of aircraft having speed V_i . We can define the average capacity of the route to mean the sets of flows $f_1, f_2, f_3 \dots$, for which the number of overtakings of aircraft per unit length of route per unit time does not exceed a specified value G_T .

As shown in Appendix D, the number of overtakings G_T is given by

$$G_T = \frac{\ell f_1}{V_1} \left[\frac{f_2(V_1 - V_2)}{V_2} + \frac{f_3(V_1 - V_3)}{V_3} + \frac{f_4(V_1 - V_4)}{V_4} + \dots \right] \\ + \frac{\ell f_2}{V_2} \left[\frac{f_3(V_2 - V_3)}{V_3} + \frac{f_4(V_2 - V_4)}{V_4} + \dots \right] \\ + \dots \quad , \quad (7)$$

where ℓ is the route length.

For any specified value of G_T and ℓ (e.g., two overtakings per hour per 100 nmi), the capacity of the air route is the set of combinations of the values of f_1, f_2, f_3, \dots that satisfies Eq. (7).

3. A Special Case

If f_1, f_2, f_3, \dots are known as fractions of the total flow f , then Eq. (7) can be rearranged in a more convenient form. Let

$$f_i = k_i f$$

Then,

$$G_T = \ell f^2 \left\{ \frac{k_1}{v_1} \left[\frac{k_2(v_1 - v_2)}{v_2} + \frac{k_3(v_1 - v_3)}{v_3} + \right] + \frac{k_2}{v_2} \left[\frac{k_3(v_2 - v_3)}{v_3} + \right] + \right\}, \quad (8)$$

$$f = \sqrt{\frac{G_T/\ell}{\phi(k_i, v_i)}} \text{ aircraft per hour}, \quad (9)$$

where $\phi(k_i, v_i)$ is equal to the expression shown in "braces" in Eq. (8).

Equation (9) gives the capacity of a single air route in terms of (G_T/ℓ) , the allowable average number of overtakings per hour per unit length of air route. The speeds of various aircraft and the fraction of the total flights for each speed are assumed to be known. (Note: It is assumed that $v_i > v_{i+1}$ without any loss of generality.)

4. Establishment of Parallel Routes

Parallel routes will be desirable whenever the flow along a route or a route section exceeds a prespecified limit based on route capacity considerations. If we let R_c indicate the allowable maximum flow of aircraft per hour along a route or a route section, then the need for and the number of parallel routes can be established by dividing the actual flow along a route by R_c . For example, let the flow along a route or route section i be a_{iL} at altitude L . Then

- If a_{iL}/R_c is less than 1, a single route is satisfactory at level L .
- If a_{iL}/R_c is greater than 1 but less than 2, two parallel routes are desirable at level L .

Similarly if a_{IL}/R_c is greater than 2 but less than 3, three parallel routes will be needed, and so on.

The spacing and the width of the routes will depend upon the quality of the available VORTAC coverage and accuracy of the navigation equipment. Suppose an air route width W nmi is established as a nominal safe route width in the region where the need for parallel route has been indicated. The center lines of the parallel routes can then be laid $(W+q)$ nmi apart with q nmi as a safety zone. The route width is discussed further later in this section.

D. Capacity of an Intersection

The expected capacity at the intersection of two routes could be defined as the maximum allowable value of the product $f_1 f_2$ (where f_1 and f_2 are the flows of aircraft per hour on the two routes) under one of the following two restrictions:

- (1) The expected number of potential conflicts per hour does not exceed a certain specified number.
- (2) The expected time that the aircraft along the two routes violate the separation minimum does not exceed a specified fraction of an hour.

1. Intersection Capacity Model under Restriction 1

From Appendix C we know that

E_c = expected number of potential conflicts per hour

$$= \frac{2f_1 f_2}{V} \times \sec \frac{\alpha}{2} \quad (10)$$

Rearranging, we get:

$$f_1 f_2 = \frac{E_c V}{2 X \sec \frac{\alpha}{2}} \quad (11)$$

Equation (11) gives the capacity at an intersection as a function of V , X , α , and some specified number of allowable potential conflicts E_c per unit time.

2. Intersection Capacity Model under Restriction 2

From Appendix C, we know that

$$H = \text{total conflict duration time per hour} = \frac{\pi X^2}{V^2} f_1 f_2 \operatorname{cosec} \alpha \quad (12)$$

Therefore,

$$f_1 f_2 = \frac{H V^2}{\pi X^2} \sin \alpha \quad (13)$$

Equation (13) gives the capacity in terms of V , X , α , and some specified allowable conflict duration time H per hour. For further details of these mathematical models, the reader is referred to Appendix D.

E. Additional Route Length per Route and Route Width

In the present air route system, some routes between community pairs are 6 to 8 percent longer than the direct great-circle route lengths. One of the reasons for such additional lengths is that the present routes are based on travel from or toward a VOR or VORTAC facility, and the location of VORTACs is frequently not in line with the direct route. In the case of RNAV routes, this restriction is not present; however, even in this case, some routes are merged or bent for ATC convenience and network simplicity. A useful attribute for route network design would be

the length added to each route because of merging or bending, obviously a simple calculation. The extra length should be kept to a minimum, since users would prefer as direct a route as possible.

Another useful attribute to compare various designs of a route network (and a VORTAC grid) would be the quality of coverage provided by the VORTACs to various routes of the network. A convenient method of estimating the quality of coverage is to compute the route widths along various routes using the route width diagram in FAA Handbook 7110.18, Air Traffic Control Service for Area Navigation Equipped Aircraft, Appendix D, page 13 (27 February 1970). Knowing the latitude and longitude of the VORTACs supporting various routes or route sections, an estimate can be made of the route widths at various portions of the route. The percentage of total route length supportable with a prespecified route width, e.g., 8 nmi, could then be used as a measure of goodness of the design of the route network and the VORTAC grid.

In Appendix I, some simplified relationships have been developed to show the percentage of a route having a route width W as a function of the orientation of the route with respect to a triangular VORTAC grid of a given side length S . The percentages are calculated for each route through a simple subprogram.

F. Total Flight Mileage

Flight mileage can be defined as the product of the length of a route in nmi and the number of flights per unit time initiated on this route. For network design, it is not sufficient to consider just route length; it is also necessary to take into account how frequently each route is used. An infrequently used route might be designed with a greater percentage of extra length than a frequently used one. This aspect of a route network can be expressed in terms of the total flight

mileage associated with the route network for some given traffic demand, and would be a significant network attribute.

The calculation of total network flight mileage is again a simple operation; it could easily be implemented through a computer program. The percentages of various routes having a route width W , as discussed above, could be multiplied by the aircraft flows along the respective routes; the percentage of flight mileage flyable with route width W could also be computed easily thereby providing a further measure of goodness of the design.

G. Congestion of Route Intersection Points

Congestion of several route intersection points in a small area is an undesirable feature, from the points of view of both the ATC controllers and the users, e.g., pilots. For the ATC controllers, it means increased surveillance and control efforts, since they have to observe several possible locations for potential conflicts. For the users, it means frequent maneuverings to avoid conflicts and congestion. A measure that indicates the location per unit area of route intersection points would be a useful network attribute. One method of evaluating this attribute would be to establish the maximum number of route intersection points in a unit area, e.g., a square area of 100-nmi side. This could be done either by observation, geometrically, or by searching the network in small steps in a systematic manner, using a computer program. A sub-program to test the total number of intersections as well as their density in a square region of specified side length is included in the computer program.

V THE DESIGN METHODOLOGY

A. The Concepts of Threads, Rope Sections, and a Primary Spider Web Network

In this section, we present some concepts agreed upon between the NAFEC and SRI team members during various working sessions in the early stages of the project. It was decided that during the experimental development of a route network design methodology, the terms "route" or "airways" should be avoided, except when these terms are used as defined in current FAR. The following concepts and definitions pertaining to routes and airways were introduced for clarity and convenience.

1. Thread

A thread is the shortest distance between two points. These points can be either the community pairs themselves or some preselected points in the vicinity of the corresponding community centers. A description of how the start and end points of a thread could be established is presented in NAFEC's Technical Note No. 8 (May 1971).*

2. Primary Thread Direction

A primary thread direction is either the direction of one of the threads connecting two community centers, or a fictitious direction from a community center (or an associated preselected point) along which

* Bibliographies, including NAFEC Technical Notes, are given at the end of each part of the report.

one or more adjacent threads will be made to lie for some distance for any of the following purposes:

- Simplifying the network
- Reducing the number of intersecting routes
- Improving safety (e.g., reducing the number of potential conflicts).

3. Rope Section

Any section of a route that consists of the combination of two or more threads is termed a rope section.

4. Primary Spider Web Network

The network resulting from connecting by threads all the community pairs exchanging traffic is referred to as the primary spider web network. Any route merging or bending will be applied to this primary spider web network. Note that in a spider web network, each thread between two communities is a great-circle path.

B. Techniques for Merging Threads to Form Rope Sections

The following ground rules and criteria were tentatively agreed upon between the NAFEC and SRI team members for the establishment of route networks, simplified on the basis of some physical and engineering considerations.

Rule 1: Where applicable, establish a few primary thread directions from a reference community center, such that the angle between two adjacent primary thread directions is at least 18° . The motivation for choosing 18° is that, typically, 50 nmi of travel from the reference community is needed for jet aircraft to obtain an altitude above 18,000 ft. At 50 nmi from the community center, the minimum angle between two threads emanating from the same community such

that the typical airways of 8 nmi width associated with the threads will cease to overlap is about 9° . Thus threads that are inclined at $\pm 9^\circ$ or less with reference to a primary thread can be treated as a part of the primary thread for the initial 50 nmi. Two primary threads that are inclined at least 18° or more from each other will be mutually exclusive and will be independent of each other.

Rule 2: After establishing a primary thread (or its direction), merge the threads in its vicinity by using one of the following approaches:

- Merge to meet ATC constraints (i.e., minimum departing angle rule).
- Merge by allowing a certain percentage mileage increase for the threads to be merged.

Rule 3: If both ends of a thread are mergeable along different primary threads, use one of the following approaches:

- Use the full length of the thread independently at each end, giving two independent one-way routes.
- Use half the length of the thread for one end and the other half for the other end.

C. Route Merging Based on Minimum Departing Angle

Referring to Rule 1 above, it was decided that, for the initial conception, threads will be merged on the basis of the minimum departing angle rule, since this rule is relevant to ATC operations and the resulting additional length is not excessive. Consider now a Community i from which Threads ij , ik , and il emanate. Any one of these could be selected as one of the primary thread directions. Suppose Thread ij has a heading of γ_j and that it is selected as a primary direction; then, by Rule 1, the other primary directions should have headings of $\gamma_j + \Delta$, $\gamma_j + 2\Delta$, ... and $\gamma_j - \Delta$, $\gamma_j - 2\Delta$, ..., and so forth. Δ is the heading increment, typically 15-20 degrees. Each of the threads emanating from Community i can be merged partially along one of the primary headings γ_j , $\gamma_j + \Delta$, ..., which are closest to the thread. The same technique

can be applied to all other communities. By selecting various combinations of reference headings, several alternative configurations can be studied and the ones that appear to have the most desirable combination of network attributes can be retained for detailed study.

In this connection, it should be noted that the heading increment Δ and the departing angle θ should be so selected that $\theta \geq \Delta/2$, otherwise the routes making an angle greater than θ but less than $\Delta/2$ with a standard heading will remain unmerged. For example, if Δ is chosen as 20° and θ as 8 degrees, a route making an angle of 10° with a standard heading will remain unmerged.

D. Assigning Altitudes

It was established during the early stages of the project that assignment of altitude is not a practicable design variable. As shown in Appendix E, the extra cost of fuel is excessive when a user is assigned an altitude other than the one he has chosen on the basis of optimizing fuel consumption and aircraft performance. Therefore, altitude assignment as a design variable was discarded. It is assumed that the user will be assigned whatever altitude he desires within the framework of the hemispherical rule unless, of course, other considerations apply, e.g., severe weather conditions. If the hemispherical rule is disregarded, possibilities of two-way air routes having the same altitude could be explored. It is proposed to study this aspect in a later phase of the project. The only practicable design variables for the present preliminary approach are thus route merging and bending, and route paralleling.

E. Summary of the Design Methodology

Below, we summarize the design methodology accomplished so far, for which a computer program has been developed and is ready in an operating form. The methodology is based on the following two policy decisions:

1. Routes will be merged using the minimum departing angle rule.
2. Eastward and westward routes will be considered separately.

The first policy decision has already been discussed. The second policy decision is based on the following considerations:

- Since the Area Navigation techniques offer the possibility of establishing independent east and west routes (it not being necessary to fly toward or away from specific VORTACS) and since the altitude bands of east and west routes are mutually exclusive, there seems to be no reason at this stage to mix the eastbound and westbound traffic and routes. It seems proper to optimize the east and west routes independently as a first step and then study other possibilities, i.e., using the same route section for both eastbound and westbound flow.
- Mixing of east and west routes would require use of more than one bend in the route when it is merged at both ends along different headings, and therefore would require a more complex programming effort. More than one bend in a route may not be desirable from other considerations also.

In view of the above, it was decided to consider the eastward and westward route networks separately in the preliminary development of the program. It should be noted that with suitable modifications, the program can be used to consider the merging of half route lengths along respective headings at each end. We now describe the various steps of the methodology:

1. Start with direct great-circle threads between each community pair:
 - Each thread will start at a point D-nmi (the assumed terminal area radius) away from one community and terminate at a point D-nmi away from another.

- Given the latitude and longitude of two points, the equation of a great-circle path is a standard algebraic expression:

$$f(\theta, \psi, \theta_1, \psi_1, \theta_2, \psi_2) = 0$$

2. Establish the intersection points between various routes:
 - A test whether two routes intersect is available.
 - The latitude and longitude of each intersection point can be calculated, as well as the angle of intersection.
3. Calculate the network attributes of interest:
 - The expected number of potential conflicts and their duration along each route per unit time.
 - The total number of expected potential conflicts and their duration.
 - The number of route intersection points in the horizontal plane and their density per unit area.
 - The total flight mileage.
 - The total extra flight mileage compared with the mileage for direct routes.
 - The maximum deviation in route length compared with the direct length.
 - Flow along various routes and route sections.
 - Flow product at various intersections and intersection capacities.
4. Select a reference route (or a reference heading) for each community and establish other standard headings from each community, using Rule 1 of Section V-B and elaborated in Section V-C. Merge the threads emanating from various communities, using the minimum departing angle rule, and compute various network attributes.
5. Repeat Step 4 by selecting other suitable reference headings for various communities.
6. Select those configurations that appear to have the most desirable combination of network attributes for detailed evaluation.

F. Applying the Methodology to Design the National Network

The methodology described above can theoretically be applied simultaneously to any number of communities and community pairs. However, the computer storage requirements and computation times would become prohibitive if excessively large numbers of community pairs are considered at the same time. Also, the human designer would find it extremely difficult to perceive and analyze the network attributes for a very large number of routes, intersections, conflicts, etc. Given both these human and computer limitations, some form of decomposition of the design process is essential.

Fortunately, the nature of traffic and community pair combinations in the United States appears to have some inherent features that can be exploited to decompose the design process into manageable parts, which can be completed separately and then combined later to produce a national network. We start with the following observations based on the study of the peak IFR data supplied to us by NAFEC.

- Total number of domestic flights per day above 18,000 ft is on the order of 11,000.
- The approximate number of community pairs (threads) exchanging various levels of traffic, and the total number of flights on these threads are shown in Table 1. The flights are also expressed as a percentage of total flights.

Table 1 indicates that a relatively small number of community pairs accounts for a major portion of the traffic. For example, if only the pairs exchanging 10 or more flights are considered, there will be 250 pairs (threads), i.e. about 11 percent of the total number of threads, and these serve more than 50 percent of the total traffic. Similarly, 500 community pairs, i.e., pairs exchanging 5 or more flights (23 percent of the threads), account for over 70 percent of the traffic. Conversely, with an increase of only 17 percent in traffic--from 72 to 89 percent--

Table 1

NUMBER OF COMMUNITY PAIRS, TRAFFIC LEVELS,
AND TOTAL FLIGHTS ABOVE 18,000 FEET

Traffic Level (exchange of flights per day)	Cumulative Number of Community Pairs (approximate)	Sum of Total Flights (approximate)	Cumulative Percentage of Total Flights
20 or more	100	4,110	37
15 or more	150	5,010	46
10 or more	250	6,210	57
5 or more	500	7,960	72
2 or more	1,200	9,800	89
1 or more	2,200	11,000	100

the number of threads has to be increased from 500 to 1,200. Thus after 500 threads, the gain in covering more traffic becomes relatively insignificant even when a large number of threads are added. It is therefore suggested that for this first phase of study, the design can be started with pairs exchanging 10 or more flights (about 250 pairs), then go on to pairs exchanging 5 or more flights, and so on.

Although only 250-500 community pairs out of 2,200 pairs account for a major portion of the high altitude traffic, these are still quite large numbers to be considered simultaneously. A possible approach to decompose the 250-500 pairs into further smaller groups is discussed below.

A study of the requested altitudes by flights between various communities has indicated that those community pairs which are less than 300-400 miles apart seldom use altitudes above 28,000 ft. On the other hand, community pairs more than about 400 miles apart seldom request

altitudes of less than 28,000 ft. Therefore, except for the number of route intersections, all other network attributes can be computed independently for the two sets of community pairs. After establishing these attributes separately for each set, the two sets can be considered simultaneously to establish only the total number of route intersections, and then the other attributes--already computed separately--can be combined algebraically to obtain a total configuration.

A preliminary list of community pairs exchanging 10 or more flights per day (above 18,000 ft) is included in Appendix H. The list is based on peak IFR data supplied to us by NAFEC. Part 1 of that list shows those pairs which are more than 400 miles apart and are expected to use basically altitudes of 28,000 ft or above. In Part 2 are listed those pairs which are less than 400 miles apart and are expected to utilize basically the altitudes between 18,000 and 28,000 ft. It is stressed that the abovementioned groupings were made considering only the high altitude flights (18,000 ft and above) and are presented as a possible preliminary approach to decomposing the community pairs for convenient implementation of the methodology. Further study of the traffic patterns and projected traffic demands might suggest other suitable groupings.

Assuming that the suggested approach is acceptable, the network for the community pairs exchanging 10 or more flights can be established in the following steps:

- (1) Consider the 125 community pairs of Part 1 in Appendix H. Using the proposed methodology and computer program, establish a few configurations that appear to have the most desirable combinations of network attributes. Let these selected configurations be called A, B, C ... etc.
- (2) Consider the community pairs in Part 2. To those pairs in which the reference community was also a reference community in Part 1, assign the standard headings corresponding to Configuration A. To other pairs, assign various suitable reference headings and establish the network attributes. Choose

that combination of reference headings which in combination with Configuration A gives the most desirable set of attributes. Let the superposition of Configuration A, along with the routes based on the best headings from set 2, be called A*. The configuration A* has all the community pairs of Part 1 and 2.

- (3) Repeat 2 except that this time assign the headings of Configuration B to the reference communities common to both sets. Establish the configuration B*.
- (4) After establishing the configurations A*, B*, C* ... by successively applying the proposed methodology and the program, compare these configurations and select the one which appears to have the best combination of attributes (and possibly has some other desirable features not included in the program).

G. Matching a Route Network and a VORTAC Grid

The route network design methodology considered so far was based on the assumption that adequate VORTAC coverage is available everywhere in the United States. At present, there exist about 280 high altitude VORTACs on the U.S. mainland and indeed these provide more than adequate coverage. However, as discussed in Part 2 of the report, adequate coverage above 18,000 ft can be provided with as few as 100 VORTACs if these are located in a suitable, orderly pattern, e.g., a triangular pattern.

One question naturally arises: would a certain orientation of a proposed triangular VORTAC grid support a given route network better than others? To answer this question, it is necessary to establish the route supporting characteristics of a VORTAC grid for various orientations of the grid. A proposed measure of the route supporting characteristics of a VORTAC grid for a given route network is the percentage of route mileage or flight mileage flyable with a certain accuracy, e.g., with a certain route width. Thus, for a given route network, if a certain orientation of a proposed triangular VORTAC grid results in higher route mileage or higher flight mileage flyable with a specified route width, e.g., 8 nmi, it can be stated that this orientation is

relatively superior to other orientations in terms of route supporting characteristics.

In Appendix I of this report, simplified relationships have been developed by which the percentage of a route flyable with a specified route width can be calculated as a function of the side length of the triangular VORTAC grid and the orientation of the route, relative to a reference row of the triangular grid.

A computer subprogram, based on the relationships shown in Appendix I, has been developed by which the percentage of route mileage and flight mileage flyable with a specified route width is computed for a given side length and orientation of a triangular grid. The percentages of route and flight mileage flyable with specified accuracies can be used as a measure of goodness of match between a route network and a VORTAC grid.

H. Establishment of Way Points and Changeover Points

After deciding upon a route network and a VORTAC grid on the basis of broad network and grid attributes, the next step would be to establish way points and changeover points along each route. The development of a detailed computer program to accomplish this task is beyond the scope of this project. However, the NAFEC team has already developed several mathematical relationships and subroutines necessary for this task. Here we only outline the steps necessary to establish the way points and changeover points with reference to NAFEC's technical notes 4 and 5 (listed in the bibliography):

- (1) Pick a route or route section *i*. The latitude and longitude of its start and end points are known.
- (2) Using the relationships of NAFEC technical note 4, find the normal distances of all the VORTACs in the vicinity of the

route under consideration. The latitude and longitude of all VORTACs are known from VORTAC grid program output.

- (3) Select those VORTACs which are at most a distance K nmi away from the route, so as to pick up only those VORTACs which can provide reasonable coverage to the route. A typical value of K could be 130 nmi.
- (4) From all those VORTACs which are at most K nmi away, pick up the best set (in some sense, e.g., giving minimum number of VORTACs). The projections of these VORTACs on the route constitute suitable way points for the route.
- (5) We now have a known set of VORTACs assured to support the route under consideration and the corresponding way points. Using the relationships of NAFEC technical note 5, establish the changeover points producing equal cross-course error.

The above outline is presented as a preliminary approach. The way points and the changeover points so established will have to be flight checked before they can be finalized.

VI COMPUTER PROGRAMS TO IMPLEMENT THE METHODOLOGY*

A. Program Section 1: To Establish Intersection Points and Angles

Note: for the derivation of the various formulas and relationships used below, refer to Appendix F.

1. Input Data

- The geodetic latitudes and longitudes of the community airports are input. A route or a route section i is specified by four numbers:

ϕ_{i1}^* = latitude at one community

θ_{i1}^* = longitude at the same community

ϕ_{i2}^* = latitude at the other community

θ_{i2}^* = longitude at the other community.

$i = 1, 2, 3, \dots n$ routes.

Convention: always choose $\theta_{i1}^* < \theta_{i2}^*$ (i.e., θ_{i1}^* always to the east of θ_{i2}^*). Distance D in nautical miles (terminal area radius).

Note: The program converts the geodetic latitudes and longitudes to geocentric system for calculating intersection points, merging, demerging, route lengths, and so on, and converts them back to geodetic units before outputting the results. Thus all outputted latitudes and longitudes are geodetic.

* The mathematical relationships described here are based on east to west network design considerations. However, only a few minor changes are necessary to make these relationships applicable to west to east network design. The necessary modifications are incorporated in the computer program.

2. What the Program is Required to Do

• Calculate the latitude and longitude (ϕ_{i1}, θ_{i1}) , (ϕ_{i2}, θ_{i2}) of the start and end points of each direct route, a distance D nmi from the community airports as follows:

First calculate:

$$d_i = \cos^{-1} [\sin \phi_{i1}^* \sin \phi_{i2}^* + \cos \phi_{i1}^* \cos \phi_{i2}^* \cos (\theta_{i2}^* - \theta_{i1}^*)]$$

then calculate:

$$\Gamma_{i1} = \cos^{-1} \left[\frac{\sin \phi_{i2}^* - \sin \phi_{i1}^* \cos d_i}{\cos \phi_{i1}^* \sin d_i} \right]$$

$$\Gamma_{i2} = \cos^{-1} \left[\frac{\sin \phi_{i1}^* - \sin \phi_{i2}^* \cos d_i}{\cos \phi_{i2}^* \sin d_i} \right]$$

then assuming $1^\circ \approx 60$ nmi:

$$\phi_{i1} = \sin^{-1} \left[\sin \phi_{i1}^* \cos\left(\frac{D}{60}\right) + \cos \phi_{i1}^* \sin\left(\frac{D}{60}\right) \cos \Gamma_{i1} \right] \quad (1)$$

$$\phi_{i2} = \sin^{-1} \left[\sin \phi_{i2}^* \cos\left(\frac{D}{60}\right) + \cos \phi_{i2}^* \sin\left(\frac{D}{60}\right) \cos \Gamma_{i2} \right] \quad (2)$$

$$\theta_{i1} = \theta_{i1}^* + \cos^{-1} \left[\frac{\cos\left(\frac{D}{60}\right) - \sin \phi_{i1} \sin \phi_{i1}^*}{\cos \phi_{i1} \cos \phi_{i1}^*} \right] \quad (3)$$

$$\theta_{i2} = \theta_{i2}^* - \cos^{-1} \left[\frac{\cos\left(\frac{D}{60}\right) - \sin \phi_{i2} \sin \phi_{i2}^*}{\cos \phi_{i2} \cos \phi_{i2}^*} \right] \quad (4)$$

- Calculate:

$$x_{i1} = \cot \phi_{i1} \cos \theta_{i1}$$

$$y_{i1} = \cot \phi_{i1} \sin \theta_{i1}$$

$$x_{i2} = \cot \phi_{i2} \cos \theta_{i2}$$

$$y_{i2} = \cot \phi_{i2} \sin \theta_{i2} \quad i = 1, 2, 3, \dots, n.$$

For $i = 1, 2, 3, \dots, n$ and for $j = (i + 1), (i + 2), (i + 3), \dots, n$, establish numbers b_{ij}

$$b_{ij} = 1 \text{ if Routes } i \text{ and } j \text{ intersect}$$

$$= 0 \text{ if Routes } i \text{ and } j \text{ do not intersect.}$$

For Routes i and j , given by:

$$\phi_{i1}, \theta_{i1}, \phi_{i2}, \theta_{i2} \quad (\text{Route } i)$$

$$\phi_{j1}, \theta_{j1}, \phi_{j2}, \theta_{j2} \quad (\text{Route } j)$$

- Calculate:

$$\lambda_{ij} = \frac{(x_{j2} - x_{i2})(y_{j2} - y_{j1}) - (x_{j2} - x_{j1})(y_{j2} - y_{i2})}{(x_{i1} - x_{i2})(y_{j2} - y_{j1}) - (x_{j2} - x_{j1})(y_{i1} - y_{i2})}$$

$$\mu_{ij} = \frac{(x_{i1} - x_{i2})(y_{j2} - y_{i2}) - (x_{j2} - x_{i2})(y_{i1} - y_{i2})}{(x_{i1} - x_{i2})(y_{j2} - y_{j1}) - (x_{j2} - x_{j1})(y_{i1} - y_{i2})}$$

Check whether $0 < \lambda_{ij} < 1$

and $0 < \mu_{ij} < 1$

If so, Routes i and j intersect; otherwise, they do not.

Print a listing of intersecting routes in a suitable format.

For all Routes i, j that intersect, find the latitude ϕ_{ij} and longitude θ_{ij} , as follows:

$$\theta_{ij} = \tan^{-1} \left[\frac{\lambda_{ij} y_{i1} + (1 - \lambda_{ij}) y_{i2}}{\lambda_{ij} x_{i1} + (1 - \lambda_{ij}) x_{i2}} \right]$$

$$\phi_{ij} = \cot^{-1} \left[\frac{\lambda_{ij} y_{i1} + (1 - \lambda_{ij}) y_{i2}}{\sin \theta_{ij}} \right]$$

Calculate the intersection angle α_{ij} of intersecting Routes i, j as follows:

First, calculate:

$$x_{ij} = \lambda_{ij} x_{i1} + (1 - \lambda_{ij}) x_{i2}$$

$$y_{ij} = \lambda_{ij} y_{i1} + (1 - \lambda_{ij}) y_{i2}$$

Then,

$$\alpha_{ij} = \cos^{-1} \left[\frac{A + B + C}{\sqrt{D E}} \right]$$

Where

$$A = (y_{ij} - y_{i1})(y_{ij} - y_{j1})$$

$$B = (x_{ij} - x_{i1})(x_{ij} - x_{j1})$$

$$C = (x_{i1} y_{ij} - y_{i1} x_{ij})(x_{j1} y_{ij} - y_{j1} x_{ij})$$

$$D = \{(y_{ij} - y_{i1})^2 + (x_{ij} - x_{i1})^2 + (x_{i1} y_{ij} - y_{i1} x_{ij})^2\}$$

$$E = \{(y_{ij} - y_{j1})^2 + (x_{ij} - x_{j1})^2 + (x_{j1} y_{ij} - y_{j1} x_{ij})^2\}$$

Print a list of intersection coordinates in a suitable format.

Print a list of intersection angles in a suitable format.

B. Program Section 2: To Establish Network Attributes

Note: for the derivation of the various formulas and expressions used below, refer to Appendices C and D.

1. Input Data

Matrix A ($n \times 5$)

a_{iL} = number of flights on Route i at altitude L during h hours

L = 1, 2, 3, 4, 5.

Listing of intersection routes and intersection angles from
Program Section 1.

Constants:

<u>Constant</u>	<u>Description</u>	<u>Typical Value</u>
r	Radar separation minima in miles	5
V	Average speed of aircraft in miles per hour	500
h	Duration of traffic count in hours	5
k	Cost per potential conflict in dollars per conflict	10
a	Allowable conflict duration in fraction of an hour	0.01
b	Allowable number of conflicts during h hours	5

2. What the Program is Required to Do

The program calculates and prints the information listed below.

a. Number of Potential Conflicts at Various Intersections

- The matrices $E_L (n \times n) (L = 1, 2, 3, 4, 5)$ with

$$c_{ij}, L = \frac{2r}{hV} b_{ij} \left(\sec \frac{\alpha_{ij}}{2} \right) a_{iL} a_{jL}$$

- The sum of the rows of $E_L (\sum_j c_{ij}, L)$ is the total number of conflicts along route i at Level L .
- The sum of the conflicts $e_L = 1/2 \sum_i \sum_j c_{ij}, L$ (the conflicts at Level L).

b. Average Conflict Duration

- The matrix $F (n \times n)$ with $f_{ij} = 15 \pi \frac{r}{V} b_{ij} \operatorname{cosec} \frac{\alpha_{ij}}{2}$.

c. Conflict Duration Time

- The $n \times n$ matrices $G_L (L = 1, 2, 3, 4, 5)$ with

$$g_{ij}, L = \frac{60 \pi r^2}{hV^2} b_{ij} (\operatorname{cosec} \alpha_{ij}) a_{iL} a_{jL}$$

- The sum of the rows of $G_L (\sum_j g_{ij}, L)$ is the total conflict duration along Route i .
- The sum of total conflict time at Level L ,

$$g_L = 1/2 \sum_i \sum_j g_{ij}, L$$

d. Intersection Capacity Based on Conflict Duration

- The matrix H ($n \times n$).

$$h_{ij} = \frac{a}{\pi} \frac{v^2}{2} b_{ij} \sin \alpha_{ij}$$

e. Intersection Capacity Based on Conflict Numbers

- The matrix M ($n \times n$),

$$m_{ij} = \frac{b}{2h} \frac{v}{r} b_{ij} \cos \frac{\alpha_{ij}}{2}$$

f. Total Flight Mileage

- The value for $\sum_i f_i \ell_i$,

where

$$f_i = \sum_L a_{iL}$$

and

$$\ell_i = 3440 \cos^{-1} [\sin \phi_{i1} \sin \phi_{i2} + \cos \phi_{i1} \cos \phi_{i2} \cos (\phi_{i2} - \theta_{i1})]$$

C. Program Section 3: Route Merging

Note: for the derivation of the various formulas and expressions used below, refer to Appendix B.

1. Input Data

The input data consist of the following:

- Input data as in Program Section 1 and specification of a reference heading or a reference route for each community.
- Constant θ , the departing angle of demerging routes.
($\approx 10^\circ$)
- Δ , heading increment, degrees (note: make sure that $\theta \geq \frac{\Delta}{2}$).

2. What the Program is Required to Do

- Let the reference direction of a community be specified as Route j.

Calculate the heading γ_j

(the angle between north and the route, measured in the clockwise direction):

First, calculate the length d_j :

$$d_j = \cos^{-1} [\sin \phi_{j1} \sin \phi_{j2} + \cos \phi_{j1} \cos \phi_{j2} \cos (\theta_{j2} - \theta_{j1})]$$

Then,

$$\gamma_j = 360^\circ - \cos^{-1} \left[\frac{\sin \phi_{j2} - \sin \phi_{j1} \cos d_j}{\cos \phi_{j1} \sin d_j} \right]$$

If the reference direction of a community is already specified as a heading j, the above calculations are not necessary.

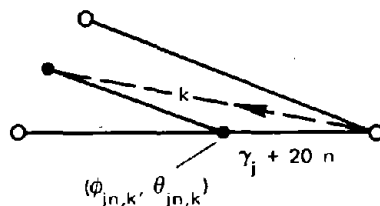
- Define standard heading directions $h_{j,n}$ ($n = 0, 1, 2, 3, \dots$):

$$h_{j,n} = (\gamma_j + \Delta n) \rightarrow \text{if } \gamma_j + \Delta n < 360^\circ$$

$$= (\gamma_j + \Delta n - 360) \rightarrow \text{if } \gamma_j + \Delta n > 360^\circ$$

All other routes from this community will be partially merged along one or the other heading directions, as shown in Figure 3 and described below.

A Route k will be merged with the heading direction for which $|h_{j,n} - \gamma_k| < \theta$. Let the latitude and longitude of the point from which route k separates be designated as $\phi_{jn,k}$, $\theta_{jn,k}$. We want $\phi_{jn,k}$ and $\theta_{jn,k}$ in terms of ϕ_{j1} , θ_{j1} and γ_j and γ_k .



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FIGURE 3 DEMERGING POINT OF A MERGED ROUTE

First, calculate:

$$g_k = d_k \{ \cos (\gamma_k - h_{j,n}) - [|\sin (\gamma_k - h_{j,n})|] \cot \theta \}$$

$$- \frac{D}{60} \{ |\sin (\gamma_k - h_{j,n})| \} \cot \theta$$

where

$$d_k = \cos^{-1} [\sin \phi_{k1} \sin \phi_{k2} + \cos \phi_{k1} \cos \phi_{k2} \cos (\theta_{k2} - \theta_{k1})]$$

(already calculated). Note that g_k is the length of the merged part.

If $g_k > 0$, then

$$\phi_{jn,k} = \sin^{-1} [\sin \phi_{j1} \cos g_k + \cos \phi_{j1} \sin g_k \cos (360 - h_{j,n})]$$

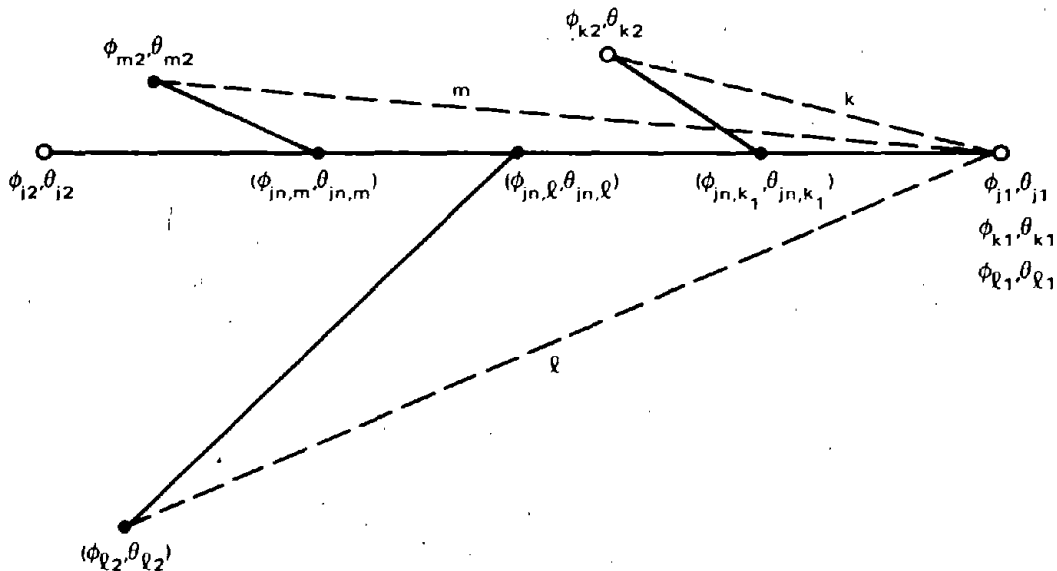
$$\theta_{jn,k} = \theta_{j1} + \cos^{-1} \left[\frac{\cos g_k - \sin \phi_{jn,k} \sin \phi_{j1}}{\cos \phi_{jn,k} \cos \phi_{j1}} \right]$$

If $g_k \leq 0$, then the corresponding route is left unmerged.

Let all routes that are mergeable with heading γ_{jn} be redesignated as $k, \ell, m \dots$, such that

$$\theta_{jn,k} < \theta_{jn,\ell} < \theta_{jn,m}$$

Then the original routes, k, ℓ, m , from the community under consideration will be redirected as shown in Figure 4.



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FIGURE 4 REDIRECTIONS OF ORIGINAL ROUTES

Route Section Between		With Flows
(ϕ_{j1}, θ_{j1})	$(\phi_{jn,k}, \theta_{jn,k})$	$s = \sum_{\ell, m, n, o} a_{sL}$
$(\phi_{jn,k}, \theta_{jn,k})$	(ϕ_{k2}, θ_{k2})	a_{kL}
$(\phi_{jn,k}, \theta_{jn,k})$	$(\phi_{jn,\ell}, \theta_{jn,\ell})$	$(\sum a_{sL}) - a_{kL}$
$(\phi_{jn,\ell}, \theta_{jn,\ell})$	$(\phi_{\ell2}, \theta_{\ell2})$	$a_{\ell L}$
$(\phi_{jn,\ell}, \theta_{jn,\ell})$	$(\phi_{jn,m}, \theta_{jn,m})$	$(\sum a_{sL}) - a_{kL} - a_{\ell L}$
$(\phi_{jn,m}, \theta_{jn,m})$	(ϕ_{m2}, θ_{m2})	a_{mL}
$(\phi_{jn,m}, \theta_{jn,m})$	(ϕ_{j2}, θ_{j2})	a_{jL}

In the original table of routes, the routes emanating from the community under consideration are modified as above. The new intersecting points are calculated by using Program Section 1. The new network attributes are calculated by using Program Section 2. The results are tabulated for study.

The entire process of the route merging program is continued by successively selecting different communities and assigning suitable reference directions to each community, establishing the corresponding standard headings, and then merging the routes from the community according to the above method until all communities are taken into account.

The designer can select various combinations of reference routes or headings and study the resulting network, compare various network attributes, and then select those configurations which appear to have the most desirable combination of attributes.

VII CONCLUDING REMARKS

The methodology and the computer program developed so far are of a preliminary nature and are based on some simplifying assumptions. The purpose of this section is to review briefly some of the significant simplifying assumptions and discuss their effects on the validity of the design methodology.

Following is a list of some important simplifying assumptions.

- The aircraft flows and speeds have been represented as average flows and speeds.
- The effects of controller judgmental factors have not been taken into account.
- Restricted areas have not been considered explicitly.
- Eastward and westward route networks are considered independently.
- The merged routes generated by the present methodology occasionally intersect with each other twice. Such situations have to be rectified manually.

We now discuss each of the above assumptions in the order stated.

A. Average Flows and Speeds

If a network were to be developed based on air traffic as it is actually distributed in time and space, one would either have to employ elaborate simulation techniques or develop statistical models requiring extensive traffic data. Both these approaches were considered beyond the scope of this project. A proposed approach to account for dynamic variations in traffic flow is to use "peak hour" flows as explained in Appendix G, producing a "conservative" network based on the assumptions that peak flows occur during the same hour. However, since in reality

this will not be the case, it will be advisable to reexamine the designed "conservative" network to ensure that no unwarranted steps were taken.

It is suggested that sample routes designed using the proposed methodology should be tested in the NAFEC simulation facilities. Such testing will provide valuable insight and will be helpful in establishing the areas of potential improvement.

B. Controller Judgment

ATC controllers play a significant role in shaping the traffic flow patterns. However, some of the actions of ATC controllers--particularly the ones based on their judgment and experience--are extremely difficult, if not impossible, to model in terms of analytical expressions. As such, human factors were not included in the preliminary design methodology. In a related project,* SRI is currently developing techniques to establish the constraints on the ATC capacity due to human controllers. It is conceivable that some of these techniques could be incorporated in the route network design methodology in a separate study.

C. Restricted Areas

In the early stages of this project, it was intended that restricted areas would be taken into account explicitly. However, it now appears that the logic and the computer storage requirements to accomplish the bending of routes around restricted areas will be too extensive to justify the cost. It is suggested that for the preliminary design, the bending of routes around the restricted areas be implemented manually, i.e., first design the routes without considering the restricted areas

* SRI Project 8181, A Methodology for Evaluating the Capacity of ATC Systems.

and then manually modify those routes which are crossing restricted areas. For example, suitable route sections around the restricted area would be selected manually and then the network attributes would be tested, using the program to select a proper modification.

D. Route Directions

The present program is meant to design the eastward and westward networks independently. The rationale for this approach has already been discussed in Section V-E.

Here we wish to point out that the two versions of the program can be used serially to merge half lengths of the routes using the eastward version and the remaining half lengths using the westward version. For example, the direct route between community pairs New York and San Francisco could be divided into two halves. The part from New York to the middle point can be merged partially along a standard heading from New York (westward direction), and the part from San Francisco to the middle point can be merged partially along a standard heading from San Francisco (eastward direction). Note that the complete route between New York and San Francisco in this approach will consist of four sections, i.e., three zigs. It will be useful to apply this approach to a sample network and to study the resulting network for comparison purposes.

E. Merged Route Intersections

In the present methodology and the associated computer program, routes are partially merged along standard headings, thereby creating a bend in the route. Thus, a route that originally consisted of a single straight line is changed into a route with two straight line sections. Occasionally, two routes i and j that originally crossed each other only once, or did not cross at all, may cross each other twice because of the bends in either or both of them, as illustrated in Figure 5.

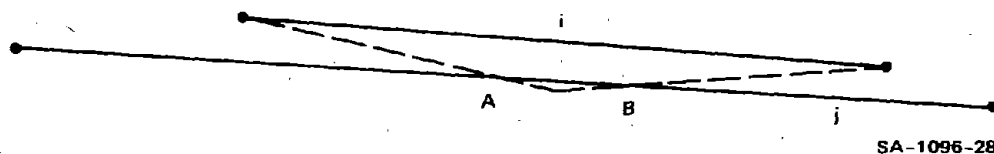


FIGURE 5 CROSSING OF A ROUTE TWICE WITH ANOTHER ROUTE BECAUSE OF THE BEND

In this figure, routes i and j were not crossing originally when direct path was provided. When route i is merged partially along some standard heading resulting in the route shown with broken lines, two intersections, A and B, result. This obviously is an undesirable situation. Rectification of such situations through computer programs would require a rather complicated logic and does not appear to be cost beneficial at this stage. It is suggested that for preliminary design purposes, these situations be manually corrected by merging the part between A and B along the route j. To assist the designer in locating such undesirable situations, a provision has been made in the program to indicate those sets of intersections and routes which must be rectified. A further check can be made by studying the plot of the network. After studying the list of all routes and intersections requiring modifications, the designer may manually specify appropriate modifications.

Although the developed methodology is not comprehensive and is based on several simplifying assumptions, it is the first systematic computer based methodology for designing large route networks. The methodology is modular in structure so that various parts can be changed or improved independently, e.g., if an improved conflict model is later developed, it can easily replace the original one without requiring any major change in the program.

Appendix A

PROBLEM DEFINITION AND GROUND RULES

This appendix describes a tentative set of ground rules for the development of the methodologies for Task 3. The rules were developed during the week beginning 26 April 1971 and reflect agreement among the NAFEC and SRI project team members on problem definition--the requisite starting point for the development of a systematic and logical network design methodology.

1. Traffic Data Base

Resolution: The IFR peak day traffic tapes will be used as the data base for representing traffic demands.

Discussion: The IFR peak day traffic tapes were chosen to provide a data base because they:

- Are frequently used for characterizing traffic levels at Air Route Traffic Control Centers (ARTCC) and Airport Traffic Control Towers (ATCT).
- Will provide a conservative (or worst-case) system design.
- Include traffic from all three classes: air carrier, military aviation, and general aviation.
- Contain most of the required traffic demand parameters at the community centers and are in a form that is easily adaptable for the present purposes.
- Can readily be scaled to represent forecasted traffic demands.

The community centers and community pairs to be included in the design of the route network should be determined on the basis of traffic level. This traffic demand specification has been incorporated into

the traffic demand model by rank ordering the community centers and community pairs according to IFR high-altitude (18,000 ft and above) traffic levels.

2. Interface with Terminal Areas

a. Departure/Arrival Points

Resolution: Departure/arrival points will be located at a radius of 50 nmi from the geometrical center of a community. Latitudes and longitudes for a set of discrete departure/arrival points will be derived and furnished for each community center in the data base.

Discussion: NAFEC will develop these points on the basis of traffic density and the angles to the interfacing communities.* Departure/arrival points derived in this manner will reflect a best choice from the aircraft user's viewpoint. In practice, geographical or wind factors may not permit this choice. Such factors will be considered in later phases of this study.

b. Climb/Descent Areas

Resolution: The problem of climb and descent is to be considered primarily as a terminal problem; it will not be handled explicitly in this phase of the design of the route network.

Discussion: Since it is known that climb to and descent from the high-altitude airspace occur beyond the 50-nmi radius that is being assumed for the terminal boundary, provisions will be made for this interface. One possible means of accomplishing this is to allot a certain

* The rule for choosing a terminal's departure/arrival points (on a circle of radius 50 nmi), which is based upon predominant (or prime) traffic flows, is described in NAFEC's Technical Note No. 8.

amount of airspace adjacent to the routes, for an additional number of miles from each departure/arrival point. This provision is a reasonable approximation for the purposes of this phase of the study, and it should provide sufficient flexibility when the problem of climb and descent in the terminal areas is considered explicitly in subsequent phases of the design.

3. Restricted Airspace

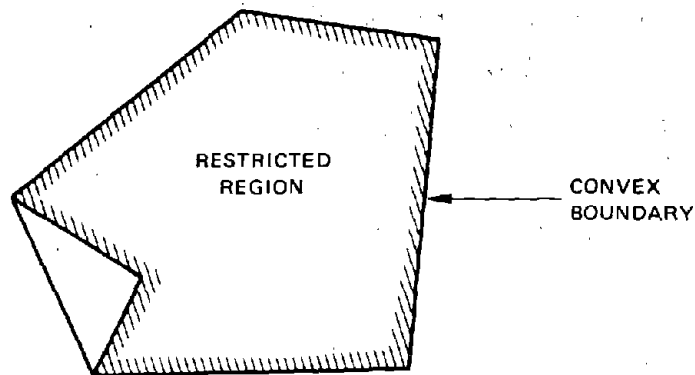
Resolution: Restricted airspace will be considered in the design of the route network.

Discussion: Since restricted airspace imposes spatial constraints (latitude, longitude, and elevation) on the location of routes, it is an important element that must be incorporated in the design of the route network structure.

The data that define restricted regions in terms of their area projection on the ground and their altitude band are readily available from navigational charts and other sources. These data are now being compiled for regions in the high-altitude (18,000 ft and above) airspace. For the purposes of high-altitude route network design, the ground projection for each restricted region can be represented by a convex boundary. This convex boundary is obtained by connecting a subset of ground projection's vertices with great circles, as illustrated in Figure A-1. It should be noted that this entails no loss of generality for the case of high-altitude routes, given the geometry and size of actual restricted regions.

4. Route Widths

Resolution: Route widths will be assumed to be constant for the design of the route network.



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FIGURE A-1 GROUND PROJECTION FOR A RESTRICTED REGION

Discussion: In practice, route width is determined by the geometrical relationship of the supporting VORTACs to the particular route. Since the design of the route network in Task 3 is to be independent of VORTAC geometry, it will be assumed that the route widths are some constant value representative of typical ATC separation requirements (e.g., 8 to 12 nmi). Furthermore, in the Task 3 methodology, route widths can be varied parametrically in the design algorithm. The relationship of route widths to VORTAC geometry will be considered explicitly in the Task 4 methodology.

5. Altitude Utilization Rules

Resolution: The hemisphere rule will be used as a starting point from which other altitude utilization rules will be investigated for possible inclusion in the Task 3 methodology.

Discussion: In the development of the Task 3 methodology, it is planned to investigate other altitude utilization rules that are potentially more efficient than the hemisphere rule in the use of available airspace. That is, by developing an algorithm that more efficiently utilizes the available altitudes, one could expect a reduction in the

number of intersections between routes or in the expected number of potential conflicts between aircraft. Such a reduction, in turn, would lead to increased capacity for the resulting route network. In this approach, the hemisphere rule would serve as a basis for comparing other altitude utilization rules.

6. Atmospheric Conditions

Resolution: Atmospheric conditions, i.e., weather, wind, and jet stream, will not be considered explicitly in the initial design of the route network.

Discussion: Too little definitive information is available to allow design to be based explicitly on atmospheric conditions. It is felt that the route network structure to be developed will be sufficiently flexible to enable the system to cope with atmospheric conditions, just as the present system does. An examination will be made in later phases of project work to determine whether this capability is implicit in the design.

7. Form of RNAV

Resolution: All aircraft will be considered to be equipped with rho-theta type RNAV equipment with slant-range correction.

Discussion: The most accurate present-day RNAV is the rho-rho system. The implementing of PVORTAC will enable rho-theta type RNAV to match the accuracy of the rho-rho type RNAV. Further, the rho-rho system requires on the order of twice the ground facilities needed in a rho-theta system. Therefore, the trend is toward the latter, and it is felt that this is the RNAV of the future. Slant-range correction will certainly be in all future systems.

8. VORTAC Coverage

Resolution: It will be assumed that full VORTAC coverage is available throughout the United States.

Discussion: The total number of VORTACs presently available in the United States is of the order of 1100 of which 300 VORTACs are meant for high altitude routes. With their present locations, the existing VORTACs provide more than adequate coverage for all practical purposes. The justification of the above assumption is thus obvious.

9. High-Altitude Level

Resolution: Only air routes altitude above 18,000 ft will be considered.

Discussion: It is planned by FAA that RNAV routes will first be introduced for high altitude routes. The altitude of 18,000 ft was chosen as a preliminary ceiling to define a suitable finite traffic data base.

Appendix B

GROUPING OF THREADS ON THE BASIS OF A MINIMUM DEPARTING ANGLE BETWEEN MERGED AND PRIMARY THREADS

The purpose of this appendix is to develop some useful relationships for the case when a minimum departing angle between a merged and a primary thread is specified. It is shown that, for a minimum departing angle of 10° , the increase in the length of the thread being merged does not exceed 0.4 percent of the direct shortest length and that, for a departing angle of 15° , the additional length is less than 0.9 percent.

1. Some Basic Relationships

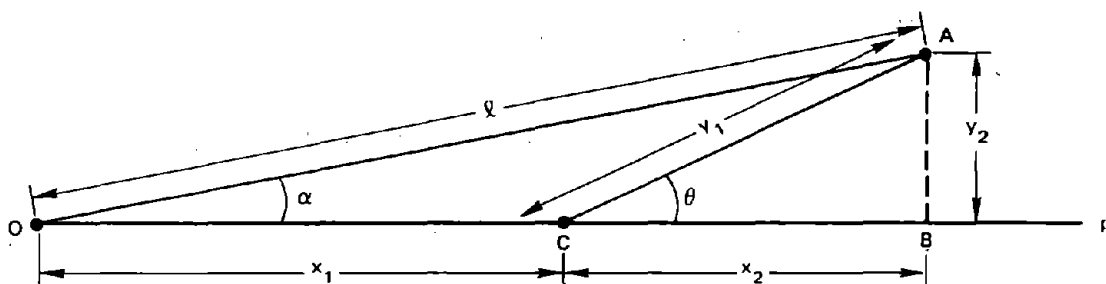
The basic relationships in the merging of threads are first established by using plane trigonometry. Later, the results are converted to a geocentric system. Referring to Figure B-1, let

OP be a primary thread

OA a candidate thread of length ℓ

Angle AOP = α

Angle θ = minimum specified departing angle



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FIGURE B-1 A PRIMARY THREAD (OP) AND A CANDIDATE THREAD FOR MERGING (OA)

OC = x_1 , length merged along the primary thread

CA = y_1 , length between the takeoff point and Destination A.

AB is perpendicular from A on OP, of length y_2 .

Length CB = x_2 .

With the above definitions, the following relationships can immediately be established:

$$y_1 \sin \theta = y_2 = \ell \sin \alpha \quad (B-1)$$

$$\therefore y_1 = \ell \frac{\sin \alpha}{\sin \theta} \quad (B-2)$$

$$\tan \theta = \frac{y_2}{x_2} = \frac{\ell \sin \alpha}{\ell \cos \alpha - x_1}$$

$$\therefore x_1 = \ell \cos \alpha - \ell \sin \alpha \cot \theta \quad (B-3)$$

$$\therefore \frac{x_1}{\ell} = \cos \alpha - \sin \alpha \cot \theta \quad (B-4)$$

Equation (B-4) gives the length of the part of the thread that is merged along the primary thread as a fraction of the original length.

Let us now calculate the increase in the length of the thread when it is partially merged. We have

$$\begin{aligned} \Delta &= \text{increase in the length} = x_1 + y_1 - \ell \\ &= \ell \cos \alpha - \ell \sin \alpha \cot \theta + \ell \frac{\sin \alpha}{\sin \theta} - \ell \\ \Delta &= \ell \left(\cos \alpha - \sin \alpha \cot \theta + \frac{\sin \alpha}{\sin \theta} - 1 \right) \quad (B-5) \end{aligned}$$

Equation (B-5) gives the increase in the length of Thread OA.

Dividing (B-5) by ℓ , we have

$$\Delta/\ell = \left(\cos \alpha - \sin \alpha \cot \theta + \frac{\sin \alpha}{\sin \theta} - 1 \right) \quad (B-6)$$

The right-hand side of (B-6) can be simplified by using standard trigonometric relations:

$$\Delta/\ell = \frac{\cos \left(\frac{\theta}{2} - \alpha \right)}{\cos \frac{\theta}{2}} - 1 \quad (B-7)$$

Equation (B-7) gives the increment in the length of the thread as a fraction of the original length.

In Table B-1, values of $\frac{x_1}{\ell} \times 100$ and $\frac{\Delta}{\ell} \times 100$ have been calculated for three fixed values of θ : 5° , 10° , and 15° .

Table B-1

RELATION BETWEEN α , x_1/ℓ , AND Δ/ℓ
FOR FIXED VALUES OF θ , THE DEPARTING ANGLE

$\theta = 5^\circ$			$\theta = 10^\circ$		$\theta = 15^\circ$	
α	$100 \frac{\Delta}{\ell}$	$100 \frac{x_1}{\ell}$	$100 \frac{\Delta}{\ell}$	$100 \frac{x_1}{\ell}$	$100 \Delta/\ell$	$100 x_1/\ell$
0.5	0.034	90	0.073	95	0.11	97
1	.061	80	.14	90	.21	93
2	.092	60	.24	80	.40	87
3	.091	40	.32	70	.55	80
5	0	0	.38	50	.77	67
7			.32	30	.86	54
9			.14	10	.83	40
12			0	0	.55	20
15			-	-	0	0

In Figure B-2, the relationships for $\theta = 10^\circ$ and 15° are shown graphically for further clarity.

From Table B-1 and Figure B-2 it is clear that, if a departing angle as large as 15° is specified, the maximum increase in the length of the thread being merged will not exceed 0.9 percent (when the thread makes an angle of about 7° with the primary thread) and that, if the acceptable departing angle is taken as 10° , then the increase in length will not be more than 0.4 percent (when the thread angle is 5°).

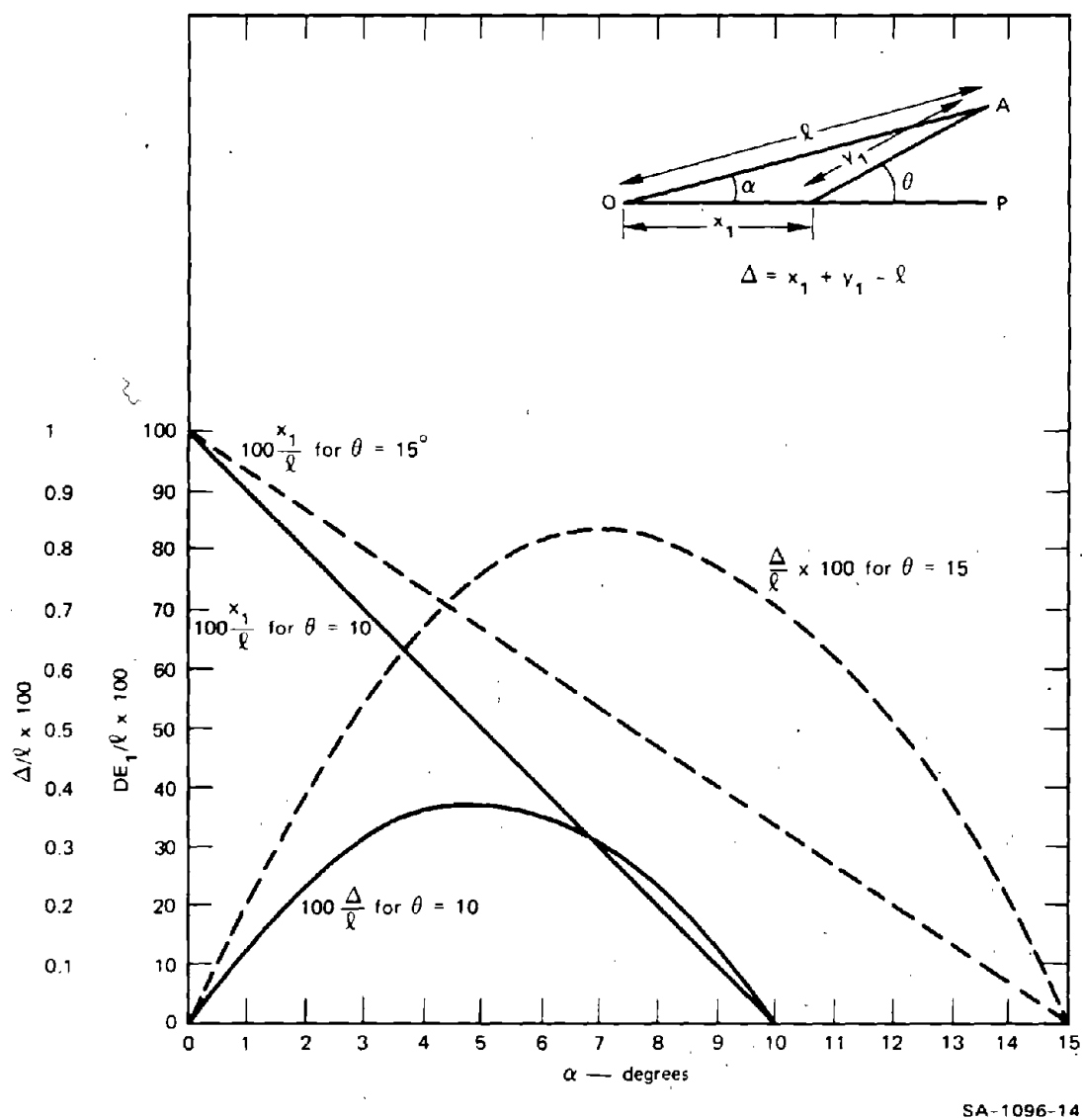


FIGURE B-2 RELATIONSHIPS BETWEEN α , Δ/l , AND x_1/l

2. The Merged Length and the Coordinates of the Take-off Point in Spherical Coordinates

Without any significant loss of accuracy, it can be assumed that Equations (B-4) and (B-7) are valid also when Points O, A, P, and C lie within a finite area of the earth's surface. Let the latitudes and longitudes of O, A, P, and C be (θ_0, ϕ_0) , (θ_1, ϕ_1) , (θ_2, ϕ_2) , and (θ_i, ϕ_i) , respectively, as shown in Figure B-3. Let N be the north pole. Let γ_1 and γ_2 be the headings (the angles with reference to local north) of Routes OA and OP. Let d_1 and d_2 be the great-circle arc angles OA and OP. Then, using standard relationships of spherical trigonometry, we have (see Figure B-3):

$$d_1 = \cos^{-1} [\sin \phi_0 \sin \phi_1 + \cos \phi_0 \cos \phi_1 \cos (\theta_1 - \theta_0)] \text{ degrees (B-8)}$$

$$d_2 = \cos^{-1} [\sin \phi_0 \sin \phi_2 + \cos \phi_0 \cos \phi_2 \cos (\theta_2 - \theta_0)] \text{ degrees (B-9)}$$

and then

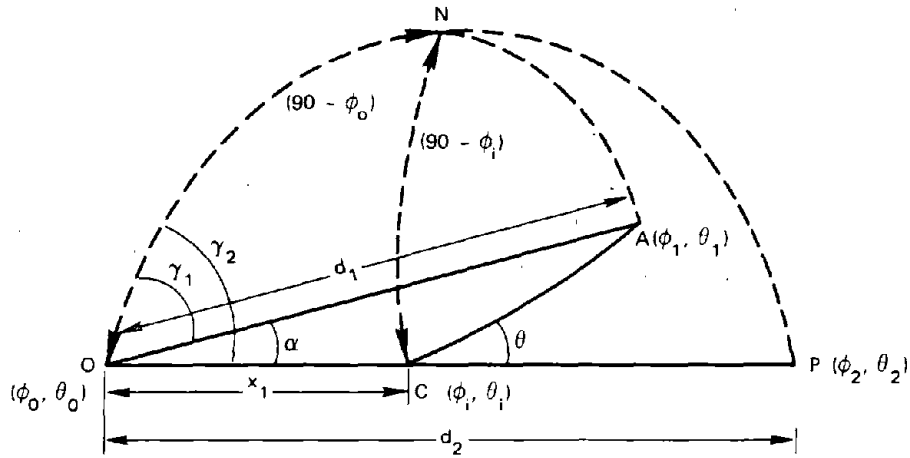
$$\gamma_1 = \cos^{-1} \left[\frac{\sin \phi_1 - \sin \phi_0 \cos d_1}{\cos \phi_0 \sin d_2} \right] \text{ degrees (B-10)}$$

$$\gamma_2 = \cos^{-1} \left[\frac{\sin \phi_2 - \sin \phi_0 \cos d_2}{\cos \phi_0 \sin d_2} \right] \text{ degrees (B-11)}$$

Now, Angle $\alpha = \angle AOP = (\gamma_1 - \gamma_2)$.

Substituting d_1 for ℓ and $(\gamma_1 - \gamma_2)$ for α in Eq. (B-4), we get

$$OC = x_1 = d_1 [\cos (\gamma_1 - \gamma_2) - \sin (\gamma_1 - \gamma_2) \cot \theta] \text{ degrees. (B-12)}$$



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FIGURE B-3 MERGED LENGTH IN SPHERICAL COORDINATES

Considering now the spherical triangle OCN, we have:

$$\cos (90 - \phi_1) = \cos (x_1) \cos (90 - \phi_0) + \sin x_1 \sin (90 - \phi_0) \cos \gamma_2 \quad (B-13)$$

or

$$\sin \phi_1 = \cos x_1 \sin \phi_0 + \sin x_1 \cos \phi_0 \cos \gamma_2$$

$$\phi_1 = \sin^{-1} \left[\sin \phi_0 \cos x_1 + \sin x_1 \cos \phi_0 \cos \gamma_2 \right] \text{ degrees.} \quad (B-14)$$

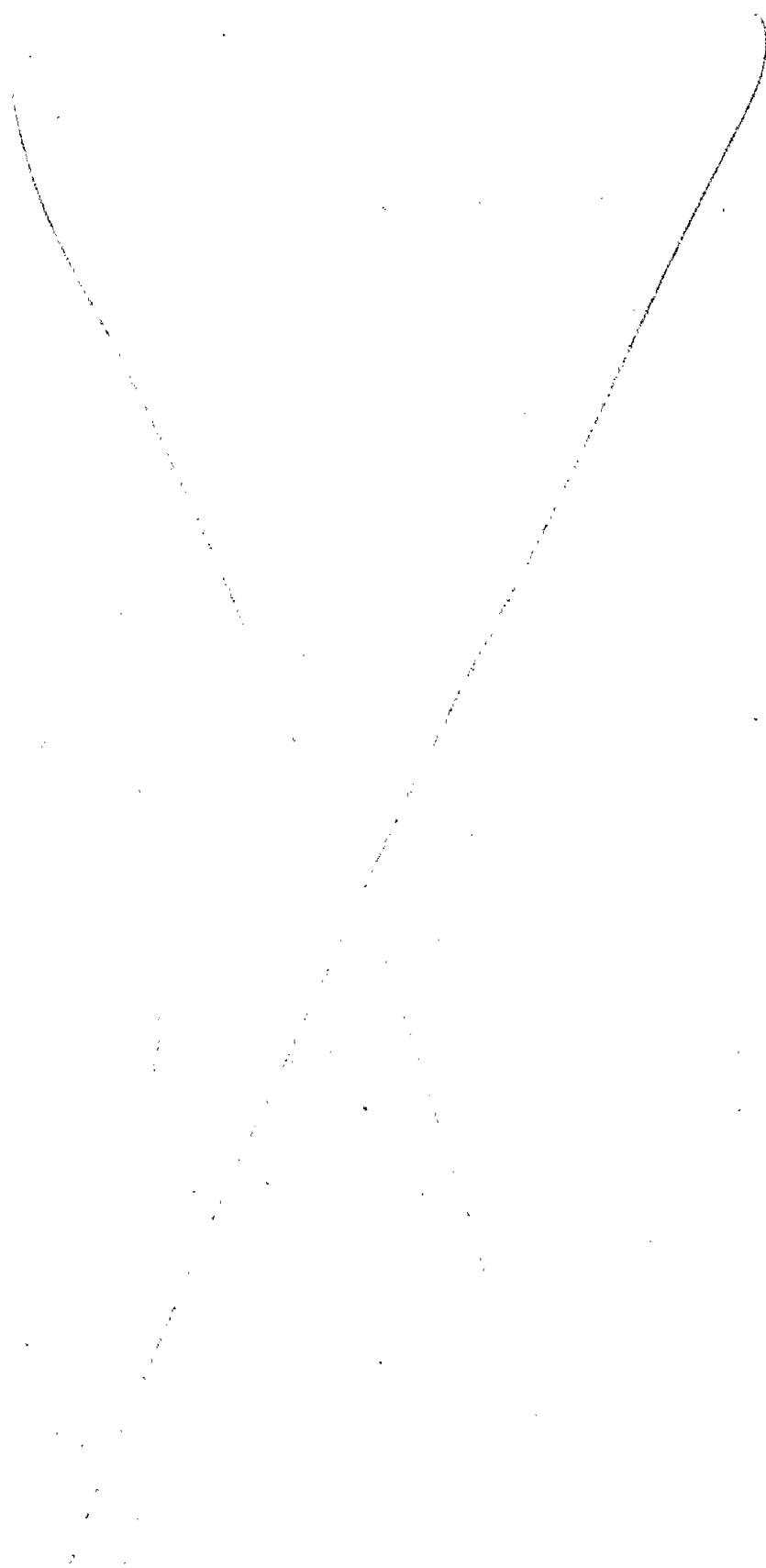
Also,

$$\begin{aligned} \cos x_1 &= \cos (90 - \phi_0) \cos (90 - \phi_1) \\ &+ \sin (90 - \phi_0) \sin (90 - \phi_1) \cos (\theta_1 - \theta_0) \end{aligned} \quad (B-15)$$

Rearranging, we get

$$\theta_i = \theta_0 + \cos^{-1} \left(\frac{\cos x_1 - \sin \phi_0 \sin \phi_i}{\cos \phi_0 \cos \phi_i} \right) \quad (\text{B-16})$$

Summarizing the results: Equation (B-12) gives the length of the merged part of a thread and Equations (B-14) and (B-16) give the latitude and longitude of the take-off point.



cc

Appendix C

CONFLICT PREDICTION MODELS

Under radar surveillance conditions, ATC is required to ensure that two aircraft are separated at least by 5 nmi. Whenever this condition is violated, the two aircraft are considered to be in conflict. By using this definition, simple mathematical models for predicting the expected number of conflicts and the conflict durations at an intersection can be developed, as shown below.

1. Development of the Mathematical Model for Expected Number of Potential Conflicts

Let:

α = angle between Routes AB and CD (see Figure C-1).

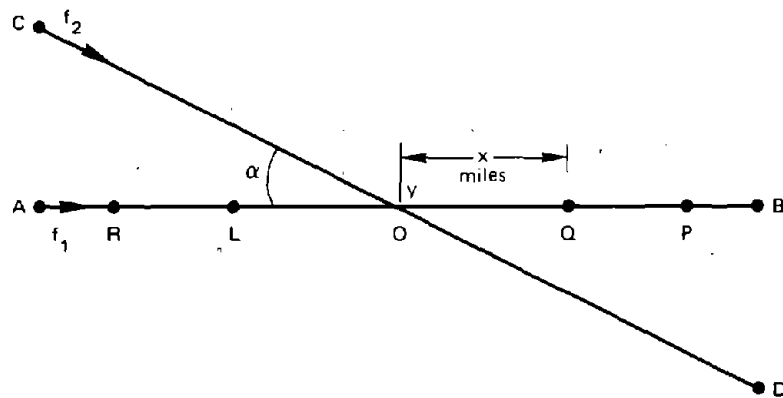
V = average velocity of aircraft on the routes (nmi per hour).

f_1 = expected flow rate along Route AB (aircraft per hour).

f_2 = expected flow rate along Route CD (aircraft per hour).

X = minimum separation allowed between two aircraft (typically, 5 nmi).

Referring to Figure C-1, let Aircraft b be at Intersection O and flying toward D. We ask ourselves where should Aircraft a not be along Route AB at this moment so that (1) a conflict between Aircraft a and b is not occurring, (2) a conflict will not develop as Aircraft a and b continue to move with velocity V along AB and CD, respectively, and



SA-1096-16

FIGURE C-1 TWO INTERSECTING ROUTES FOR EVALUATION OF POTENTIAL CONFLICTS

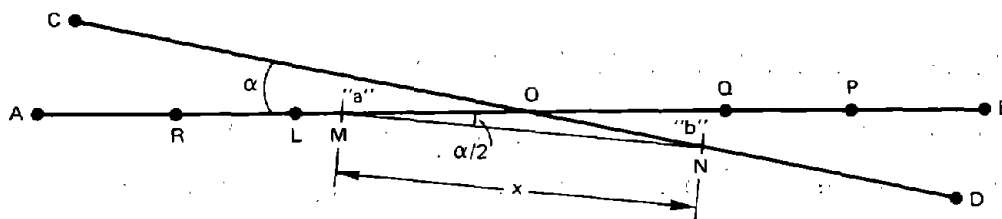
(3) a conflict would not have occurred in the past if Aircraft b and a were moved backwards. We denote such portions as critical length along AB.

- Consider first the portion of the route to the left of O. Let L be a point such that $OL = X$. If Aircraft a is between O and L, it is now in conflict with Aircraft b. However, if it is beyond (to the left of) L, it is not now in conflict with b and could not have been in conflict in the past, as can easily be checked by moving each aircraft backward. However, as b flies toward D and a flies toward O, the distance between them becomes shortest when the distance of a from O along \overrightarrow{AO} is equal to the distance of b from O along \overrightarrow{OD} . (This fact can be proved either mathematically or by simple geometric trials.) We must make sure that this shortest distance is $\geq X$. Let R be the nearest point to O such that Aircraft a being at this point when Aircraft b is at O, will come at most no closer than X miles to b, as both aircraft continue their flight. Now, referring to Figure C-2, let a be at M and b at N; when the aircraft reach these points,

$$OM = ON \text{ (condition for shortest distance)}$$

But since the aircraft are assumed to travel with the same speed, the distance RM covered by a should be equal to ON--the distance covered by b in the same time period. Thus,

$$RM = ON = OM$$



SA-1096-17

FIGURE C-2 SHORTEST DISTANCE BETWEEN AIRCRAFT a AND b

By assumption, $MN = X$ miles. Now,

$$\frac{(OM)}{1/2(MN)} = \sec \frac{\alpha}{2}$$

$$OM = 1/2(MN) \sec \frac{\alpha}{2}$$

$$OR = MN \sec \frac{\alpha}{2} = X \sec \frac{\alpha}{2}$$

Therefore, the critical length to the left of O is equal to $X \sec \alpha/2$.

- Consider now the portion of the route to the right of O. Let Q be a point X miles from O. If an aircraft a is somewhere between O and Q, it is now in conflict with b. If it is beyond Q, then it is not now in conflict with b and cannot come into conflict in future. However, considering the past and applying a reasoning similar to that used to calculate the length OR on the left-hand side for the future, a length OP equal to OR exists on the right-hand side such that Aircraft a in the strip QP has been in conflict with Aircraft b, as can be checked by moving Aircraft a now between Q and P, and Aircraft b, now at O, backward. Thus the critical length on the right-hand side is also $X \sec \alpha/2$. Denoting the length RP by L_c , the critical length, we have:

$$L_c = 2X \sec \frac{\alpha}{2} \text{ nmi}$$

a. Expected Number of Potential Conflicts

Every aircraft of flow f_1 along AB will occupy the critical length L_c for a period of $\frac{L_c}{V}$ hours. Therefore, during a period of 1 hour, the critical length L_c will be occupied by flow f_1 for a period of $f_1 \frac{L_c}{V}$ hours. The number of aircraft along CD arriving at the intersection during this period will be $f_1 \frac{L_c}{V} f_2$. It is not difficult to see that $f_1 \frac{L_c}{V} f_2$ is the expected number of potential conflicts per hour. Denoting by E_c the expected number of potential conflicts per hour, and substituting for L_c , we get

$$E_c = 2X \frac{f_1 f_2}{V} \sec \frac{\alpha}{2} \text{ conflicts per hour.} \quad (C-1)$$

The above expression indicates that, for fixed values of X , f_1 , f_2 , and V , the expected number of conflicts increases as α increases.

b. Saturation Flow Rate

An interesting interpretation of the above-noted relationship is as follows:

Let $f_1 \geq f_2$. This entails no loss of generality. For fixed values of α and X , the value of f_1 that makes the expression $2X f_1/V \sec \alpha/2$ equal to unity will block the intersection continuously and can be regarded as the saturation flow rate at the intersection. With this flow rate on one route, every aircraft on the crossing route will experience a conflict.

Thus, the saturation flow rate at an intersection is:

$$f_s = \frac{V}{2X} \cos \frac{\alpha}{2} \quad (C-2)$$

c. Conflict Intensity Index

A convenient method to characterize the conflict characteristics of an intersection would be to normalize Equation (C-1) with reference to the product $f_1 f_2$. Thus, defining $E/f_1 f_2$ as the conflict intensity index, I , we have

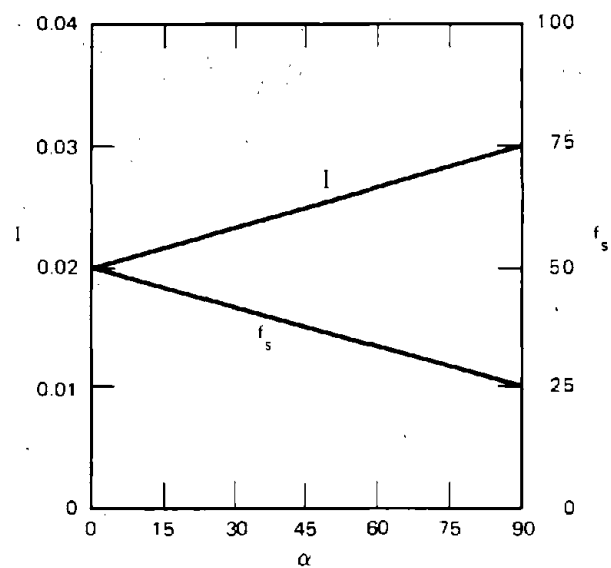
$$I = E_c / f_1 f_2 = \frac{2X}{V} \sec \frac{\alpha}{2} \quad (C-3)$$

Note that $I = 1/f_s$. For any specified values of X and V , a normalized graph indicating the relationships between I and α and f_s and α can be established. These curves can be used as a convenient reference to calculate the number of predicted conflicts when α , f_1 , and f_2 are known.

In Figure C-3, the relationship between $f_s \sim \alpha$ and $I \sim \alpha$ is shown for $X = 5$ nmi and $V = 500$ mph.

2. Mathematical Model for Conflict Duration

The potential conflict prediction model based on radar separation rules, as described above, gives only the number of potential conflicts to be expected per unit time but does not give any indication about the expected duration of these conflicts. The purpose of the discussion below is to develop an analytical expression giving the expected duration of conflicts as a function of the intersecting angle α and the rates of flow along the intersecting routes. It is shown that, for the lower values of α , the number of conflicts is less but the expected duration of the conflicts is longer, compared with the higher values of α , for which the number of conflicts is greater but the expected duration of the conflicts is shorter. Some relationships between conflict duration and intersection capacity are also developed.

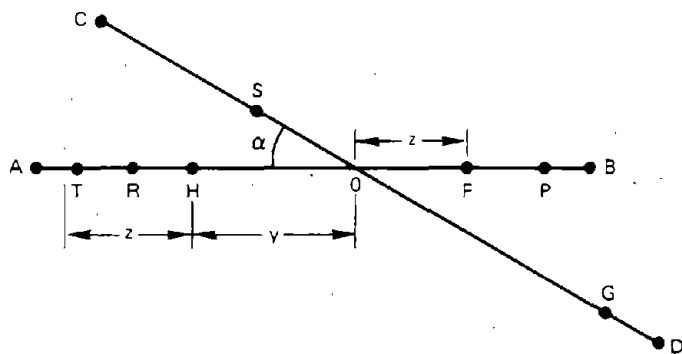


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FIGURE C-3 CONFLICT INTENSITY INDEX AND SATURATION FLOW RATE AS A FUNCTION OF α , ASSUMING $X = 5$ MILES AND $V = 500$ mph

a. Expected Time in Conflict

Consider Routes AB and CD intersecting at an angle α , as shown in Figure C-4. Suppose Aircraft b is now at O traveling toward D.



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FIGURE C-4 TWO INTERSECTING ROUTES FOR EVALUATION OF CONFLICT DURATION

Let $OR = OP = X \sec \alpha/2$,

Where X = the radar separation minimum.

We know (from Section 2) that if Aircraft a along AB is anywhere in RP, then one of the following is true in relation to Aircraft b:

- It is now in conflict
- It has been in conflict
- It will be in conflict.

If Aircraft a is outside the section RP, then no conflict between Aircraft a and b is occurring, has occurred, or will occur. We concern ourselves with the condition when b is at O and a is somewhere in RP. Our assumption is that any point along RP is an equally likely one for Aircraft a to be. Thus, consider a point H a distance of y miles away from O, and assume that Aircraft a is at H when Aircraft b is at O. The duration of conflict in this situation will be the period starting from the time Aircraft a and Aircraft b first come within X nmi of each other until the time when the distance ceases to be less than or equal to X nmi.

Let both Aircraft b and a be moved back in time, so that b is at S and a is at T, such that $OS = TH = Z$. The largest value of Z such that the length TS is just X will be the situation when a and b first entered into potential conflict situation. From then onward the distance between a and b continued to be less than X until a reached the point F and b reached G, with $OF = Z$ and $OG = Y + Z$. Thus the duration of conflict is the time spent in traveling the distance $Y + Z + Z = 2Z + Y$; i. e., the conflict duration time is $\frac{2Z + Y}{V}$ hours.

We now develop an expression for the length $2Z + Y$. Referring to Figure C-4, let Z be such that $ST = X$ miles; then,

$$(Y + Z)^2 + (Z)^2 - 2(Y + Z)Z \cos \alpha = X^2, \quad (C-3)$$

or

$$(2 - 2 \cos \alpha) Z^2 + (2Y - 2Y \cos \alpha) Z + (Y^2 - X^2) = 0 \quad (C-4)$$

or

$$Z^2 + YZ + \frac{(Y^2 - X^2)}{2(1 - \cos \alpha)} = 0$$

or

$$Z^2 + YZ + \frac{(Y^2 - X^2)}{4 \sin^2 \frac{\alpha}{2}} = 0$$

$$Z = \frac{-Y \pm \sqrt{Y^2 - \frac{Y^2 - X^2}{\sin^2 \frac{\alpha}{2}}}}{2}$$

(C-5)

$$= \frac{-Y \pm \operatorname{cosec} \frac{\alpha}{2} \sqrt{X^2 - Y^2 \cos^2 \frac{\alpha}{2}}}{2}$$

We choose the plus sign, since a negative value of Z would indicate going forward in time, the aspect we have already taken into account by adding a length Z to the right of O.

Thus .

$$Z = \frac{-Y + \operatorname{cosec} \frac{\alpha}{2} \sqrt{X^2 - Y^2 \cos^2 \frac{\alpha}{2}}}{2}$$

$$\text{and } 2Z + Y = \text{conflict duration distance} = \operatorname{cosec} \frac{\alpha}{2} \sqrt{X^2 - Y^2 \cos^2 \frac{\alpha}{2}}$$

$$= \cot \frac{\alpha}{2} \sqrt{X^2 \sec^2 \frac{\alpha}{2} - Y^2} \quad (C-6)$$

The expected conflict duration distance D is given by

$$D = \frac{1}{X \sec \frac{\alpha}{2}} \int_0^{X \sec \frac{\alpha}{2}} \cot \frac{\alpha}{2} \sqrt{X^2 \sec^2 \frac{\alpha}{2} - Y^2} dy \quad (C-7)$$

$$= \frac{\cot \frac{\alpha}{2}}{X \sec \frac{\alpha}{2}} \left\{ \frac{1}{2} \left[Y \sqrt{X^2 \sec^2 \frac{\alpha}{2} - Y^2} + X^2 \sec^2 \frac{\alpha}{2} \sin^{-1} \left(\frac{Y}{X \sec \frac{\alpha}{2}} \right) \right]_0^{X \sec \frac{\alpha}{2}} \right\}$$

$$= \frac{\cot \frac{\alpha}{2}}{2 X \sec \frac{\alpha}{2}} \left(\frac{\pi}{2} X^2 \sec^2 \frac{\alpha}{2} \right),$$

$$D = \frac{\pi}{4} X \operatorname{cosec} \frac{\alpha}{2} \quad (C-8)$$

Assuming an average speed of V mph, the expected conflict duration time per conflict τ is

$$\tau = \frac{\pi}{4} \frac{X}{V} \operatorname{cosec} \frac{\alpha}{2} \text{ hours.} \quad (C-9)$$

b. Conflict Duration Time per Hour

Multiplying the expected number of conflicts by τ gives the total time (as a fraction of an hour) per hour during which aircraft along Routes AB and CD would remain in conflict if nothing were done by ATC. Thus,

$$\begin{aligned}
 H = \text{conflict time duration per hour} &= \frac{\pi}{4} \frac{X}{V} \operatorname{cosec} \frac{\alpha}{2} \frac{2 f_1 f_2}{V} X \sec \frac{\alpha}{2} \\
 &= \pi \frac{X^2}{V^2} f_1 f_2 \frac{1}{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} \\
 H &= \pi \frac{X^2}{V^2} f_1 f_2 \operatorname{cosec} \alpha \quad . \quad (C-10)
 \end{aligned}$$

Let us define conflict duration index h to be $= H/f_1 f_2$, so

$$h = \pi \frac{X^2}{V^2} \operatorname{cosec} \alpha \quad . \quad (C-11)$$

c. Some Typical Numerical Results

Let $X = 5$ nmi and $V = 500$ mph. With these values, the values of τ and h are calculated for various values of α in Table C-1.

For the design of a route network, it appears that the number of potential conflicts alone may not be a sufficiently complete attribute. The time duration of the conflicts should also be considered. It is evident that, even though the number of conflicts increases with α , the average conflict duration time per hour decreases as α increases.

Table C-1

AVERAGE CONFLICT DURATION AND CONFLICT DURATION
INDEX AS A FUNCTION OF α

α	τ [hours (minutes)]	$h = \pi \frac{x^2}{v^2} \operatorname{cosec} \alpha$
5	0.183 (11)	.0036
10	.09 (5.4)	.0018
30	.03 (1.8)	.00063
60	.016 (0.94)	.00036
90	.011 (0.67)	.00031

85

Appendix D

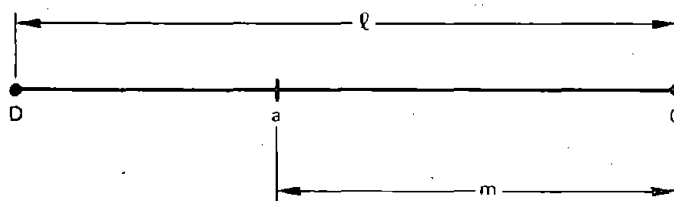
AIR ROUTE AND INTERSECTION CAPACITY MODELS

1. Capacity of a Single Air Route when no Other Route Intersects and no Passing is Permitted

Suppose that the speeds of aircraft using a common air route vary between V_1 and V_2 , V_1 being $> V_2$. Let the route length be ℓ . Let the separation minimum be X miles; i.e., two aircraft, one behind the other, should not come closer than X miles at any time. The worst condition in terms of one aircraft following another is when an aircraft flying at V_2 mph is followed by an aircraft flying at V_1 . So that the faster aircraft will not overtake the slower aircraft before the slower has reached the destination D , the faster aircraft must be released from the origin O at a time T hours after the slower aircraft, where T has to have a certain minimum value. We first calculate this value.

Referring to Figure D-1, let Aircraft a be a distance m miles away from O after T hours, when Aircraft b is released from O . Now

$$m = TV_2 \quad (D-1)$$



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FIGURE D-1 A SINGLE AIR ROUTE FOR CALCULATING OVERTAKING TIME

If Aircraft a reaches D when Aircraft b is X miles away from D, no conflict will ensue, since b will already have reached its destination.

Thus we must have

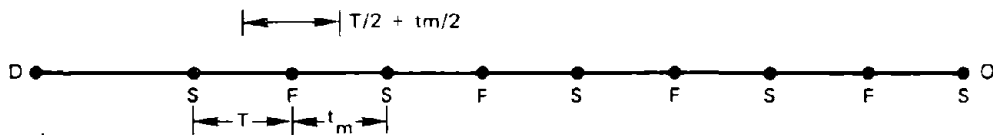
$$\frac{\ell - m}{V_2} = \frac{\ell - X}{V_1} \quad (D-2)$$

$$\frac{\ell - TV_2}{V_2} = \frac{\ell - X}{V_1} \quad (D-3)$$

$$T = \frac{\ell}{V_2} - \frac{(\ell - X)}{V_1} \quad (D-4)$$

$$= \frac{\ell V_1 - V_2 (\ell - X)}{V_1 V_2} \quad (D-5)$$

If more than two aircraft follow each other during a period of, say, 1 hour, the worst sequence in terms of one aircraft overtaking another would be as shown in Figure D-2. We denote by S the slowest and by F



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FIGURE D-2 WORST SEQUENCE OF SLOW AND FAST AIRCRAFT

the fastest aircraft. Theoretically a slow aircraft following a fast aircraft can be released from the origin immediately after the release of the other. However, according to ATC rules, there has to be a lapse of a certain minimum time t_m before the release of the second aircraft.

Thus, in Figure D-2, going from right to left, the time separation between S and F is t_m and that between F and S is T. It can easily be seen that there is one aircraft in each of the time spans $(T/2 + t_m/2)$. Therefore, the average number of aircraft that can be released along Route OD is

$$\left(\frac{2}{t_m + T} \right) \text{ aircraft per hour}$$

$$= \frac{2}{t_m + \frac{[\ell V_1 - V_2 (\ell - X)]}{V_1 V_2}} \text{ aircraft per hour} \quad (D-6)$$

Let us denote this number of aircraft as A_{\min} , the minimum capacity of the air route:

$$A_{\min} = \frac{2}{t_m + \left[\frac{\ell}{V_2} - \frac{(\ell - X)}{V_1} \right]} \text{ aircraft per hour} \quad (D-7)$$

Here A_{\min} is the minimum theoretical capacity, assuming the worst sequence of slow and fast aircraft when no passing is allowed. Actually, the worst sequence is very unlikely to occur, and passing of aircraft is allowed. A more realistic measure of capacity is considered in the concluding remarks on this model.

2. Some Typical Numerical Results

Let

$$t_m = 2 \text{ minutes} = 1/30 \text{ hour}$$

$$X = 20 \text{ miles}$$

$$V_1 = 300 \text{ mph.}$$

Then,

$$A_{\min} = \frac{2}{\frac{1}{30} + \frac{\ell}{V_2} - \frac{(\ell - 20)}{600}}$$

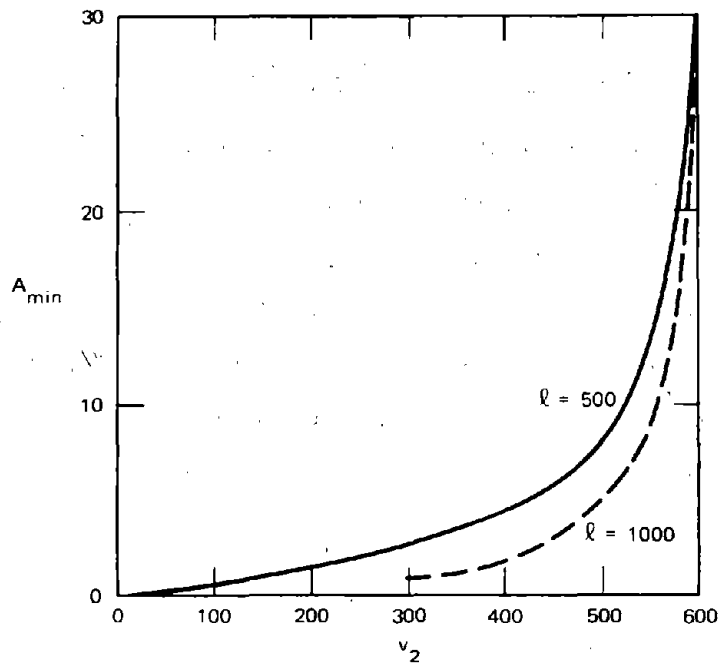
Values of A_{\min} for several values of V_2 are shown for $\ell = 500$ and 1,000 below. These values are also plotted in Figure D-3.

$$A_{\min} \sim V_2 \quad (V_1 = 600 \text{ mph})$$

V_2	A_{\min}	
	$\ell = 500 \text{ miles}$	$\ell = 1,000 \text{ miles}$
600	30	30
550	13	9
500	8	5
400	5	2
300	3	1

3. Concluding Remarks on the Capacity Model with No Passing Allowed

A study of Figure D-3 indicates that the theoretical minimum capacity of a single route, when no passing is allowed, decreases very rapidly even when the speed of two aircraft differs only slightly. Thus, with both V_1 and V_2 being 600 nmi, the capacity is 30 aircraft per hour. With V_2 being 550 nmi (i.e., only 8 percent less than the fast speed), the capacity is reduced to only 13, less than half. These numbers also indicate the amount of workload on ATC to implement overtaking of aircraft if overtaking is allowed. It is easily inferred that slight variation in the speeds of aircraft will significantly increase the ATC workload associated with providing passing guidance.



SA-1096-22

FIGURE D-3 VALUES OF A_{min} FOR SEVERAL VALUES OF V_2

A more realistic measure of capacity would be to consider the typical mix of aircraft, assuming suitable distribution, and to establish a capacity measure, possibly on the basis of allowing a certain number of passing aircraft per unit length of route per unit time, as discussed below.

4. A Proposed Definition of the Average Capacity of a Single Route with Passing Allowed

Let a single route be used by various types of aircraft having speeds V_1, V_2, V_3, \dots mph, V_i being greater than V_{i+1} . Let f_i be the flow per hour of the aircraft having speed V_i . We define the average capacity of the route to mean the sets of flows f_1, f_2, f_3, \dots for which the number of overtaking aircraft per unit length of route per unit time does not exceed a specified value.

5. Derivation of the Route Capacity Model with Passing Allowed

Let us first consider only two types of aircraft, having speeds V_1 and V_2 , with $V_1 > V_2$. Let the corresponding flows be f_1 and f_2 per hour. Consider a route length of ℓ miles. If we consider flow f_2 independently, the average spacing between aircraft with speed V_2 will be V_2/f_2 . The speed of the fast aircraft relative to the slow aircraft is $(V_1 - V_2)$ mph. During a period of 1 hour, the string of fast-moving aircraft will travel a distance of $(V_1 - V_2)$ miles relative to the string of slow-moving aircraft. During this period, each fast-moving aircraft will overtake $(V_1 - V_2)f_2/V_2$ slow-moving aircraft. In the length ℓ there will be, on the average, $\ell f_1/V_1$ fast-moving aircraft. Thus, during a period of 1 hour, the total number of overtakings G will be

$$G = \frac{\ell f_1 f_2 (V_1 - V_2)}{V_1 V_2} \quad (D-8)$$

If there were other types of aircraft having speeds of V_3, V_4, \dots with $V_1 > V_2 > V_3, \dots$, the total number of overtakings per hour would be:

$$\begin{aligned} G_T = & \frac{\ell f_1}{V_1} \left[\frac{f_2 (V_1 - V_2)}{V_2} + \frac{f_3 (V_1 - V_3)}{V_3} + \frac{f_4 (V_1 - V_4)}{V_4} + \dots \right] \\ & + \frac{\ell f_2}{V_1} \left[\frac{f_3 (V_2 - V_3)}{V_3} + \frac{f_4 (V_2 - V_4)}{V_4} + \dots \right] \\ & + \dots \quad (D-9) \end{aligned}$$

For any specified value of G_T and ℓ (e.g., two overtakings per hour per 100 miles), the capacity of the air route is the set of all the combinations of the values of f_1, f_2, f_3, \dots that satisfies Eq. (D-9).

6. An Example

Let us consider the simple case of two types of aircraft with $V_1 > V_2$. Letting $\ell = 500$ miles, we have

$$G = \frac{500 f_1 f_2 (V_1 - V_2)}{V_1 V_2} \quad (D-10)$$

If G equals one overtaking allowed per hour, then

$$f_1 f_2 = \frac{V_1 V_2}{500 (V_1 - V_2)}$$

gives the capacity of the air route per 500 miles in terms of the product of the two types of aircraft. Let $V_1 = 600$; then the values of $f_1 f_2$ for various values of V_2 are as shown in the following table:

V_2	$f_1 f_2$	
	$\ell = 500$ miles	$\ell = 250$ miles
550	13	26
500	6	12
450	3	7

Note: For $V_1 = V_2$, we must use the capacity based on the minimum allowable separation, e.g., if the separation minimum is X , the capacity is V/X aircraft per hour.

7. A Special Case of Capacity Calculations with Passing Allowed

If f_1, f_2, f_3, \dots are known as fractions of the total flow f , then Eq. (D-9) can be rearranged in a more convenient form. Let $f_i = k_i f$; then,

$$G_T = \ell f^2 \left\{ \frac{k_1}{V_1} \left[\frac{k_2 (V_1 - V_2)}{V_2} + \frac{k_3 (V_1 - V_3)}{V_3} + \dots \right] + \frac{k_2}{V_2} \left[\frac{k_3 (V_2 - V_3)}{V_3} + \dots \right] + \dots \right\} \quad (D-11)$$

$$f = \sqrt{\frac{G_T / \ell}{\phi(k_i, V_i)}} \quad \text{aircraft per hour} \quad , \quad (D-12)$$

where $\phi(k_i, V_i)$ is equal to the expression in braces in Eq. (D-11). Equation (D-12) gives the capacity of a single air route in terms of (G_T / ℓ) , which is the allowable average number of overtakings per hour per unit length of air route. The speeds of various aircraft and the fraction of the total flights for each speed are assumed to be known. (Note: It is assumed that $V_i > V_{i+1}$ without any loss of generality.)

8. An Example for the Special Case

Let us consider a mix of three types of aircraft:

$$\text{Let } v_1 = 600 \quad v_2 = 500 \quad v_3 = 400$$

$$\text{Let } k_1 = 0.6 \quad k_2 = 0.2 \quad k_3 = 0.2$$

$$\text{Let } G_T/l = 1/250 \text{ i.e., one overtaking per 250 mph.}$$

Substitution of the assumed values in Eq. (D-12) would give

$$f = 5 \text{ aircraft per hour}$$

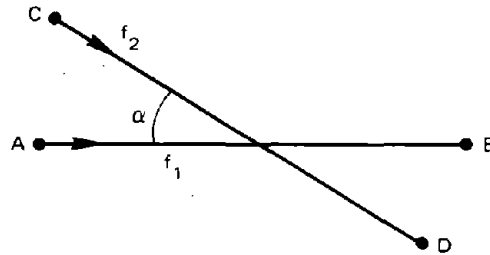
9. Concluding Remarks on Capacity Model with Passings Allowed

A definition of average air route capacity based on a certain allowable number of overtakings per hour per unit length of air route has been presented above. If the mix of aircraft of various speeds is not known, the capacity is a set of various values of flows f_1, f_2, f_3, \dots fulfilling a certain constraint (Eq. D-9). If the mix or fraction of aircraft of various speeds is known, capacity can be calculated as a single number (Eq. D-12). Two examples have been presented to illustrate the concepts. These concepts will be pursued further and any new approaches reported in due course.

10. Capacity at an Intersection

Referring to Figure D-4, a possible definition of an aircraft capacity at the intersection of two routes would be the maximum allowable value of the product $f_1 f_2$ (where f_1 and f_2 are the flows of aircraft per hour on the two routes) under one of the following restrictions:

- (1) The expected number of potential conflicts per hour does not exceed a certain specified number.
- (2) The expected time the aircraft along the two routes spend in violation of the separation minimum does not exceed a specified fraction of an hour.



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FIGURE D-4 TWO INTERSECTING ROUTES FOR CAPACITY EVALUATION

11. Intersection Capacity Model under Restriction One

From Appendix C, we know that

$$\begin{aligned}
 E_c &= \text{expected number of potential conflicts per hour} \\
 &= \frac{2f_1 f_2}{V} \times \sec \frac{\alpha}{2}
 \end{aligned}
 \tag{D-13}$$

Rearranging, we get

$$f_1 f_2 = \frac{E_c V}{2 \times \sec \frac{\alpha}{2}}
 \tag{D-14}$$

Letting $E_c = 1$, i.e., one conflict per hour allowed, we have

$$f_1 f_2 = \frac{V}{2X} \cos \frac{\alpha}{2}
 \tag{D-15}$$

Equation (D-15) gives the capacity at an intersection as a function of average aircraft speed V , separation minimum X , and intersection angle, α , under the restriction that one conflict per hour is acceptable on the average. The capacity is directly proportional to the allowable number of conflicts per hour.

Assuming typical values for V and X to be 600 and 5, respectively, Table D-1 gives values for capacity as a function of intersection angle α . These values are plotted in Figure D-5 for convenience.

Table D-1

INTERSECTION CAPACITY AS A FUNCTION OF INTERSECTION ANGLE
(One Conflict per Hour Acceptable)

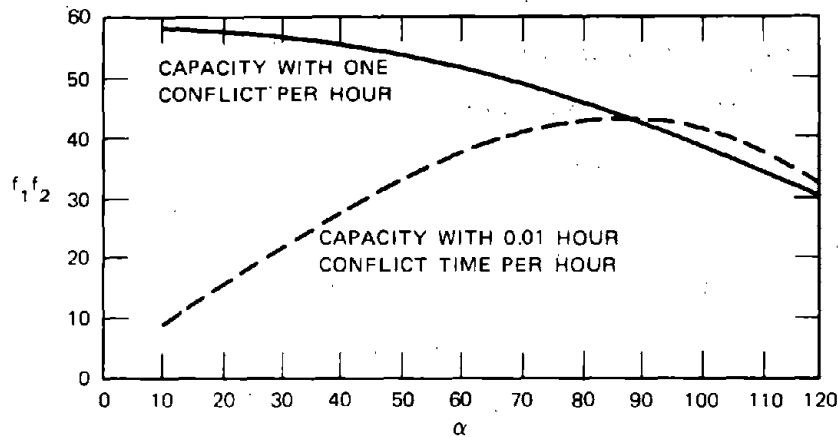
Intersection Angle α (degrees)	Capacity $f_1 f_2$ (unit conflict per hour)
10	59
20	59
30	58
45	55
60	51
90	42
120	30

12. Intersection Capacity Model under Restriction Two

From Appendix C we have

$$H = \text{conflict duration time per hour} = \frac{\pi X^2}{V^2} f_1 f_2 \operatorname{cosec} \alpha \quad (\text{D-16})$$

$$\therefore f_1 f_2 = \frac{HV^2}{\pi X^2} \sin \alpha$$



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FIGURE D-5 INTERSECTION CAPACITY AS A FUNCTION OF α

Letting $H = 0.01$, i.e., the conflict duration time per hour is 1 percent per unit time, we get

$$f_1 f_2 = 0.01 \frac{V^2}{\pi X^2} \sin \alpha \quad (D-17)$$

In Table D-2, values of $f_1 f_2$ as a function of α are tabulated for $V = 600$ and $X = 5$. The results are also plotted in Figure D-5.

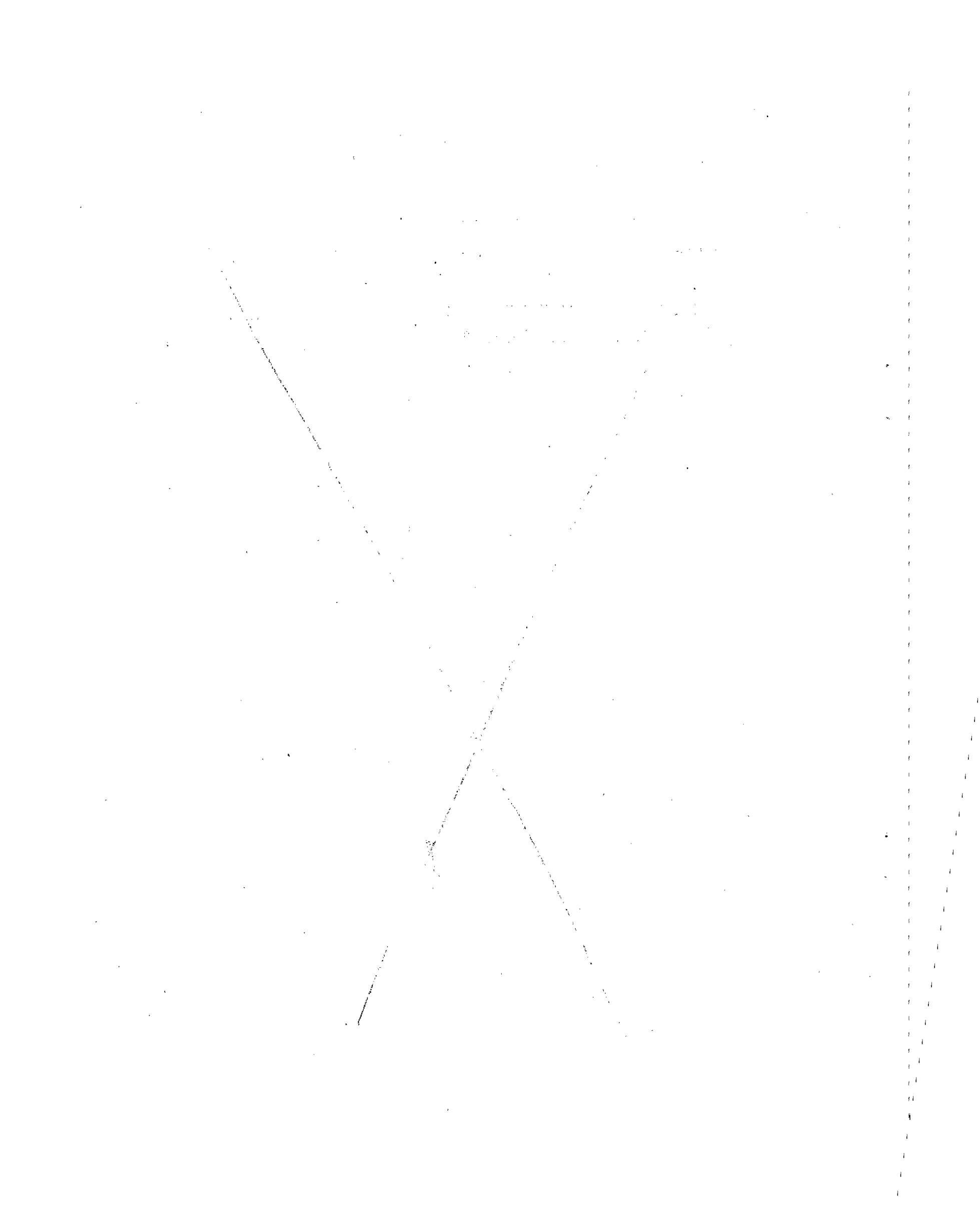
13. Concluding Remarks on Intersection Capacity

A study of Figure D-5 indicates the interesting fact that, for $\alpha < 90^\circ$, the intersection capacity based on conflict number decreases slowly as α increases, whereas the capacity based on conflict duration time increases as α increases. For $\alpha > 90^\circ$, both capacities decrease as α increases. Further study appears to be necessary to establish which capacity definition corresponds best to real world situations.

Table D-2

INTERSECTION CAPACITY AS A FUNCTION OF INTERSECTION ANGLE
(1-percent Time Allowed in Conflict)

Intersection Angle α (degrees)	Capacity $f_1 f_2$ (for 1-percent conflict duration)
10	8
30	23
45	32
60	39
90	45
120	39
150	23



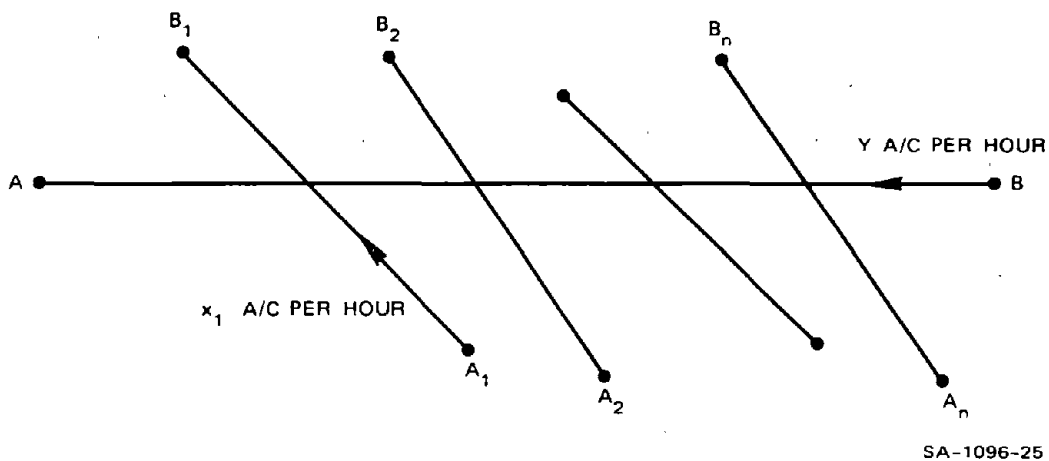
Appendix E

ASSIGNING AN ALTITUDE OTHER THAN THE DESIRED ONE

The purpose of this appendix is to show that, within the framework of the hemispherical rule, altitude assignment is not a practicable design variable.

1. Consequence of Assigning Altitudes Other than that Desired by the User

Consider a Route AB such that it intersects Routes A_1B_1 , A_2B_2 , ..., A_nB_n at the common altitude level (e.g., 31,000 ft), as shown in Figure E-1.



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FIGURE E-1 SEVERAL INTERSECTING ROUTES FOR ALTITUDE ASSIGNMENT

A possible method of removing the intersection is to assign to Route AB an altitude other than that used by Routes A_1B_1 , A_2B_2 , ..., and so on. Frequently, however, it is desirable for a route such as

AB to have the same altitude as that of A_1B_1 , A_2B_2 , ..., on the basis of optimum fuel consumption rate and so forth. Thus, if flights along AB were assigned an altitude one level lower or higher, the fuel costs would be increased.

a. Fuel Costs

Let the increase in fuel cost be $\$K_1$ per 100 miles if an altitude other than that desired is used. If the length of Route AB is ℓ , and the flow is Y aircraft per hour, the extra fuel cost C_1 for using a nonoptimum flight level will be:

$$C_1 = K_1 Y \frac{\ell}{100} \text{ dollars per hour.} \quad (E-1)$$

b. Conflict Costs

If the desired altitude is used, this being the same as that used by A_1B_1 , A_2B_2 , ..., then potential conflicts will result. If x_1, x_2, x_3, \dots is the flow of aircraft per hour along A_1B_1, A_2B_2, \dots the number of expected potential conflicts per hour E_c is

$$E_c \approx \frac{Y}{V} (x_1 + x_2 + x_3 + \dots) X \sec \frac{\alpha}{2} \quad (E-2)$$

where

V = average aircraft speed (miles per hour).

X = separation minimum

α = average angle of intersection.

Assuming $V = 500$, $X = 5$, and $\sec \alpha/2 \approx 1$, we get

$$E_c \approx \frac{Y(x_1 + x_2 + x_3, \dots)}{50} \quad (E-3)$$

Let K_2 be the cost per conflict in dollars, then C_2 , the cost of conflicts per hour, is

$$C_2 = \frac{K_2 Y}{50} (x_1 + x_2 + x_3 + \dots) \quad (E-4)$$

Considering costs C_1 and C_2 , it is clear that, if $C_1 < C_2$, then it is advantageous to assign an altitude other than that desired; otherwise it is not--i.e., for using undesired altitudes the condition is

$$K_1 Y \frac{\ell}{100} < \frac{K_2}{50} Y (x_1 + x_2 + x_3, \dots) \quad (E-5)$$

Cancelling Y on both sides and rearranging,

$$\frac{K_1}{K_2} < \frac{2(x_1 + x_2 + \dots)}{\ell} \quad (E-6)$$

or

$$\frac{K_1}{K_2} < \frac{\text{twice the sum of flights per hour on intersecting routes}}{\text{length of candidate route for altitude modification}} \quad (E-7)$$

c. An Example

We now consider a typical case. The direct route from New York to Los Angeles intersects 13 routes in the first 1,000 miles of its length at the altitude of 35,000 ft. The sum of the flights on the 13 routes at 35,000 ft is typically 50 flights during 5 hours of peak traffic, assuming that the peak traffic occurs in the same hours (actually, the sum will be less than 50 because of diversity in peaking). This gives about ten flights per hour on the intersecting routes. If,

on the basis of these figures, the route from New York to Los Angeles is to be assigned an altitude of, say, 39,000 ft for the first 1,000 miles, thereby removing all intersections (there is no cross traffic at this level), we must have

$$\frac{K_1}{K_2} < \frac{2 \times 10}{1000}$$

$$< \frac{1}{50}$$

i.e.,

$$\frac{\text{Cost per 100 nmi of the altitude change}}{\text{Cost per conflict}} < \frac{1}{50}$$

It has been estimated that for a jet aircraft, the cost per 100 nmi of an altitude change is about \$10. Thus, under present traffic conditions, unless the cost of a conflict were \$500 or more, it would not be profitable to change the altitude of the New York to Los Angeles route from 35,000 to 39,000 ft (for the first 1,000 nmi). Similar remarks are obviously applicable to other routes.

2. Consideration of Future Traffic

If the traffic increases by a factor of n , the number of conflicts increases by a factor of n^2 . Therefore, assuming an n -fold increase in traffic, the relationships (E-5) and (E-6) will be modified to

$$K_1 n Y \frac{\ell}{100} < \frac{K_2 n^2 Y}{50} (x_1 + x_2 + x_2 + \dots) \quad (\text{E-8})$$

or

$$\frac{K_1}{K_2} < \frac{2n(x_1 + x_2 + x_3 \dots)}{\ell} \quad (\text{E-9})$$

Considering the route from New York to Los Angeles again, we have

$$\frac{K_1}{K_2} < \frac{n}{50}$$

Letting $K_1 = \$10$ and $K_2 = \$50$ per conflict, we get

$$\frac{10}{50} < \frac{n}{50} ,$$

or n would have to be 10 or greater before it would be profitable to change the altitude of the New York to Los Angeles route from 35,000 to 39,000 ft. Similar remarks are applicable to other routes.

The justification of assigning an altitude other than that desired is a saving in costs. The calculations for a typical route indicate that, with assumed costs of

\$10 per 100 miles of flight at an undesired altitude, and
\$50 per conflict,

the present traffic intensity would have to increase at least tenfold for altitude change to be profitable. Thus, the principle may be applicable to traffic conditions some time in the far future, but it does not appear to be of much advantage now, unless the conflict cost increases significantly.

Appendix F

THE COORDINATES AND INTERSECTION ANGLE OF ROUTE INTERSECTING POINTS

1. Necessary and Sufficient Conditions for Intersection of Two Route Segments

a. Conditions in a Plane

Let two route segments, i and j , in a plane be defined by their end points (x_{i1}, y_{i1}) , (x_{i2}, y_{i2}) and (x_{j1}, y_{j1}) , (x_{j2}, y_{j2}) , as shown in Figure F-1.

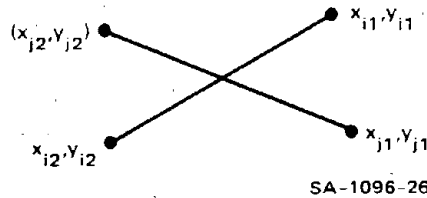


FIGURE F-1 TWO ROUTE SEGMENTS IN A PLANE

Let a general point on these segments be expressed as

$$\left. \begin{aligned} x_n &= \lambda_{ij} x_{i1} + (1 - \lambda_{ij}) x_{i2} \\ y_n &= \lambda_{ij} y_{i1} + (1 - \lambda_{ij}) y_{i2} \end{aligned} \right\} \quad (F-1)$$

$$\left. \begin{aligned} x_m &= \mu_{ij} x_{j1} + (1 - \mu_{ij}) x_{j2} \\ y_m &= \mu_{ij} y_{j1} + (1 - \mu_{ij}) y_{j2} \end{aligned} \right\} \quad (F-2)$$

where λ_{ij} and μ_{ij} are two independent constants. If, for some values of λ_{ij} and μ_{ij} such that

$$0 \leq \lambda_{ij} < 1 \quad \text{and} \quad 0 < \mu_{ij} < 1,$$

we can obtain:

$$x_n = x_m \quad \text{and} \quad y_n = y_m,$$

then this common point will be the intersecting point of the two route segments. If no such values of λ_{ij} and μ_{ij} exist, then the routes do not intersect.

Now, solving for λ_{ij} and μ_{ij} by using Cramer's Rule, we have:

$$\lambda_{ij} = \frac{\begin{vmatrix} (x_{j2} - x_{i2}) & (x_{j2} - x_{j1}) \\ (y_{j2} - y_{i2}) & (y_{j2} - y_{j1}) \end{vmatrix}}{\begin{vmatrix} (x_{i1} - x_{i2}) & (x_{j2} - x_{j1}) \\ (y_{i1} - y_{i2}) & (y_{j2} - y_{j1}) \end{vmatrix}}; \quad \mu_{ij} = \frac{\begin{vmatrix} (x_{i1} - x_{i2}) & (x_{j2} - x_{i2}) \\ (y_{i1} - y_{i2}) & (y_{j2} - y_{i2}) \end{vmatrix}}{\begin{vmatrix} (x_{i1} - x_{i2}) & (x_{j2} - x_{j1}) \\ (y_{i1} - y_{i2}) & (y_{j2} - y_{j1}) \end{vmatrix}} \quad (F-3)$$

If the solution of (F-3) results in:

$$0 < \lambda_{ij} < 1$$

$$0 < \mu_{ij} < 1,$$

then the route segments intersect; otherwise, they do not. This provides us with a necessary and sufficient condition for route intersections in a plane. A corresponding relationship using spherical coordinates is established below.

b. Conversion to Spherical Coordinates

Let the end points of the two route sections, i and j, be given on the earth's surface in terms of latitude and longitude as follows:

Route i: ϕ_{i1} = latitude at one end of Route i
 θ_{i1} = longitude at one end of Route i
 ϕ_{i2} = latitude at the other end of Route i
 θ_{i2} = longitude at the other end of Route i

Route j: ϕ_{j1} , θ_{j1} , ϕ_{j2} , θ_{j2} correspondingly.

These coordinates can be transformed to plane coordinates by using gnomonic projection, which transforms great-circle routes to straight lines. Also, since the condition of intersection is independent of the coordinate system, the Equation (F3) can be used to establish whether the two routes intersect. The gnomonic transformation gives the following relationships:

$$\left. \begin{aligned} x_{i1} &= \cot \phi_{i1} \cos \theta_{i1} \\ y_{i1} &= \cot \phi_{i1} \sin \theta_{i1} \\ x_{i2} &= \cot \phi_{i2} \cos \theta_{i2} \\ y_{i2} &= \cot \phi_{i2} \sin \theta_{i2} \end{aligned} \right\} \text{ for Route i (F-4)}$$

and x_{j1} , y_{j1} , x_{j2} , y_{j2} correspondingly. We can use these values in Equation (F3) to calculate λ_{ij} and μ_{ij} and thus establish whether Routes i and j intersect.

2. Coordinates of the Intersection Point

If the two routes intersect, the latitude ϕ_{ij} and longitude θ_{ij} of the intersection point can be calculated as follows:

Let

$$x_{ij} = \cot \phi_{ij} \cos \theta_{ij}$$

$$y_{ij} = \cot \phi_{ij} \sin \theta_{ij}$$

be the gnomonic projections of the intersection point. From Equation (F-1) we have

$$\left. \begin{aligned} x_{ij} = x_n = x_m &= \cot \phi_{ij} \cos \theta_{ij} = \lambda_{ij} x_{i1} + (1-\lambda_{ij}) x_{i2} \\ y_{ij} = y_n = y_m &= \cot \phi_{ij} \sin \theta_{ij} = \lambda_{ij} y_{i1} + (1-\lambda_{ij}) y_{i2} \end{aligned} \right\}, \quad (F-5)$$

which gives

$$\tan \theta_{ij} = \left[\frac{\lambda_{ij} y_{i1} + (1-\lambda_{ij}) y_{i2}}{\lambda_{ij} x_{i1} + (1-\lambda_{ij}) x_{i2}} \right]$$

$$\theta_{ij} = \tan^{-1} \left[\frac{\lambda_{ij} y_{i1} + (1-\lambda_{ij}) y_{i2}}{\lambda_{ij} x_{i1} + (1-\lambda_{ij}) x_{i2}} \right], \quad (F-6)$$

$$\phi_{ij} = \cot^{-1} \left[\frac{\lambda_{ij} y_{i1} + (1-\lambda_{ij}) y_{i2}}{\sin \theta_{ij}} \right]. \quad (F-7)$$

3. Angle of Intersection

Referring to Figure F-2, let C be the center of a sphere representing the earth. Consider two intersecting routes, i and j, with ϕ_{ij} , θ_{ij} as the latitude and longitude of the intersection point, as calculated above. The angle α_{ij} between the two intersecting routes is the same as the angle between the normals to the two planes, one containing the points (ϕ_{i1}, θ_{i1}) , (ϕ_{ij}, θ_{ij}) , and C and the other containing the points (ϕ_{j1}, θ_{j1}) , (ϕ_{ij}, θ_{ij}) , and C.

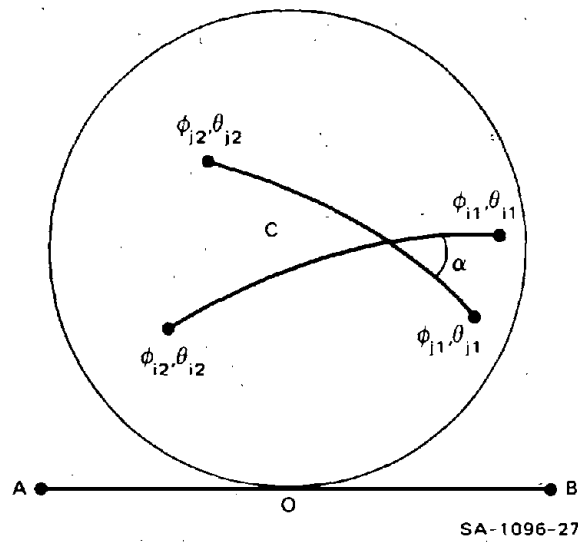


FIGURE F-2 INTERSECTION ANGLE BETWEEN TWO GREAT-CIRCLE ROUTES

Since the angle between the planes under consideration is not dependent upon the magnitude of the radius of the sphere, let the radius be assumed to be unity for convenience. Define a tangent plane AOB as the $Z = 0$ plane on which gnomonic projections will be considered. Let the tangent point O be the origin, so that the coordinates of C are $(0,0,1)$.

For the gnomonic projections, the two planes mentioned above can be expressed in determinant form as follows:

- Plane 1 containing (ϕ_{i1}, θ_{i1}) , (ϕ_{ij}, θ_{ij}) , and C:

$$\begin{vmatrix} x & x_{i1} & x_{ij} & 0 \\ y & y_{i1} & y_{ij} & 0 \\ z & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0, \quad (\text{F-8})$$

where x_{i1} and y_{i1} are given by (F-4), and x_{ij} and y_{ij} are given by (F-5).

- Plane 2 containing (ϕ_{j1}, θ_{j1}) , (ϕ_{ij}, θ_{ij}) , and C:

$$\begin{vmatrix} x & x_{j1} & x_{ij} & 0 \\ y & y_{j1} & y_{ij} & 0 \\ z & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0. \quad (\text{F-9})$$

The normals to the above planes have components proportional to the cofactors of x , y , and z in the above determinants. Thus, the normal vectors are given by:

$$X_i = \left(\begin{vmatrix} y_{i1} & y_{ij} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} x_{i1} & x_{ij} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} x_{i1} & x_{ij} & 0 \\ y_{i1} & y_{ij} & 0 \\ 1 & 1 & 1 \end{vmatrix} \right) \quad (\text{F-10})$$

and

$$X_j = \left(\begin{vmatrix} y_{j1} & y_{ij} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} x_{j1} & x_{ij} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} x_{j1} & x_{ij} & 0 \\ y_{i1} & y_{ij} & 0 \\ 1 & 1 & 1 \end{vmatrix} \right). \quad (\text{F-11})$$

Now, by using standard relationships from vector analysis, the angle α_{ij} between these two vectors X_i and X_j is given by:

$$\alpha_{ij} = \cos^{-1} \left(\frac{X_i \cdot X_j}{\|X_i\| \|X_j\|} \right), \quad (F-12)$$

which expands to:

$$\alpha_{ij} = \cos^{-1} \left[\frac{(y_{ij} - y_{i1})(y_{ij} - y_{j1}) + (x_{ij} - x_{i1})(x_{ij} - x_{j1})}{\sqrt{\{(y_{ij} - y_{i1})^2 + (x_{ij} - x_{i1})^2 + (x_{i1}y_{ij} - y_{i1}x_{ij})^2\}}} \right. \\ \left. \frac{+ (x_{i1}y_{ij} - y_{i1}x_{ij})(x_{j1}y_{ij} - y_{j1}x_{ij})}{\sqrt{\{(y_{ij} - y_{j1})^2 + (x_{ij} - x_{j1})^2 + (x_{j1}y_{ij} - y_{j1}x_{ij})^2\}}} \right]. \quad (F-13)$$

Since $x_{i1}, y_{i1} \dots$ have already been computed, the above expression is easy to calculate.

Appendix G

ESTABLISHING THE TRAFFIC DEMAND MATRIX A

The purpose of this appendix is to describe how the traffic demand matrix A, needed as an input to the computer program, can be established from the existing Peak IFR data tapes.

The Peak IFR demand data tapes have been arranged in various formats by NAFEC. Two of the tapes, suitable for establishing the demand matrix A, are the following:

- (1) The tape containing a listing of the number of aircraft per day requesting various altitudes between various communities.
- (2) The tape containing the count of aircraft between various communities for each hour during 24 hours (without any reference to the altitude).

From the first tape, the number of aircraft desiring various altitudes can be computed as fractions of total traffic flow between the respective communities. For example, let the number of aircraft desiring altitudes of 28,000, 31,000, and 35,000 ft between communities i and j be 2, 4, and 4 respectively. If these three are the only altitudes under consideration, then:

A fraction $\frac{2}{2 + 4 + 4} = 0.2$ of the total flow of 10 A/C
desires the altitude 28,000 ft,
0.4 desires the altitude 31,000 ft,
and 0.4 desires the altitude 35,000 ft.

From the second tape, the highest hourly count of traffic between i and j can be easily found. If this highest count is multiplied by

the various fractions calculated above, we will obtain the number of aircraft desiring various altitudes during the peak hour traffic activity. Thus, if the highest hourly count between i and j is 5, then it can be inferred that:

- 1 aircraft desires the altitude 28,000 ft,
- 2 aircraft desire the altitude 31,000 ft,
- 2 aircraft desire the altitude 35,000 ft.

Similar calculations can be performed for other community pairs and the demand matrix A established for the desired routes.

Study of the actual data indicated that the hourly counts are typically 0, 1, or 2 aircraft per hour. Multiplying the highest count of 2 with various fractions would frequently give the aircraft flow per hour at various altitudes in fractions, i.e., the demand matrix A will contain fractions. There is nothing wrong with fractional aircraft flows as far as the computation of network attributes is concerned. However, to make the demand matrix contain only integer numbers, a convenient method is to multiply the fractions with a suitable factor h , e.g., 5 and round off the numbers. The numbers thus resulting can be interpreted as the demand during h hours. In the preliminary test runs, a value of 5 was used for h .

Appendix H

A PRELIMINARY LIST OF COMMUNITY PAIRS EXCHANGING 10 OR MORE IFR FLIGHTS PER DAY, USING ALTITUDES OF 18,000 FEET OR ABOVE

The list of community pairs that starts on the next page was prepared on the basis of the 1969 Peak IFR data supplied to SRI by NAFEC. The "Enroute IFR Air Traffic Survey" peak day fiscal year 1968, published by the FAA's Office of Management Services was also used as a guide.

The list has been arranged so that the reference communities are always to the east of the paired communities. Only the pairs within US48 are considered.

The paired communities are listed in two parts. In Part 1 are those communities that are more than 400 miles apart and whose traffic is expected to employ basically altitudes of 28,000 ft or above. In Part 2 are listed those communities that are less than 400 miles apart and are expected to employ essentially altitudes between 18,000 ft and 28,000 ft. Both parts thus apply only to flights at or above 18,000 ft.

Part 1. Pairs employing essentially altitudes of 28,000' or above .

(Exchange of 10 or more flights per day)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
1	JFK	MDW	713	194
2		MIA	1092	94
3		DET	482	66
4		LAX	2451	55
5		SFO	2571	47
6		CLE	403	38
7		ATL	748	35
8		STL	875	34
9		DAL	1374	23
10		MSP	1018	19
11		HOU	1420	17
12		CMH	570	16
13		NEW	1171	16
14		PBI	1030	13
15		TPA	1005	13
16		DEN	1631	12
17		MEM	962	12
18		SDF	660	12
19		CLT	533	11
20		IND	646	11
21		SEA	2400	11
22	MDW	LAX	1745	80
23		SFO	1858	69
24		DEN	920	60

Part 1 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
25	MDW	MKC	414	60
26		DAL	803	38
27		SEA	1737	28
28		LAS	1524	20
29		OMA	432	20
30		PHX	1453	17
31		PDX	1758	14
32		HOU	937	12
33		TUL	598	12
34		MEM	482	10
35	DET	LAX	1983	17
36		MSP	543	13
37		SFO	2090	13
38		ATL	596	10
39	HOU	LAX	1370	16
40	MIA	MDW	1188	37
41		ATL	604	36
42		NEW	669	17
43		CLE	1087	13
44		DET	1152	13
45		PIT	1010	13
46		STL	1070	10

Part 1 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
47	CLE	LAX	2049	14
48		STL	492	13
49		ATL	554	10
50	BOS	MDW	851	45
51		MIA	1255	22
52		LAX	2596	18
53		DET	613	16
54		CLE	551	15
55		PIT	494	14
56		SFO	2699	10
57	SEA	SFO	678	44
58	DCA	MDW	597	43
59		ATL	543	20
60		MIA	923	18
61		LAX	2300	17
62		STL	712	17
63		DAL	1185	13
64		BNA	450	12
65		MSP	930	10
66		TYS	430	10

Part 1 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
67	DEN	LAX	831	38
68		SFO	949	24
69		SEA	1020	16
70		PHX	586	16
71		LAS	606	12
72		PDX	985	12
73	DAL	LAX	1240	38
74		SFO	1483	24
75		ELP	572	18
76		DEN	663	12
78	PHX	LAS	418	30
79		SFO	653	20
80	PIT	MDW	410	38
81		ATL	521	17
82	PHL	MDW	666	40
83		DET	443	23
84		MIA	1019	23
85		ATL	666	22
86		LAX	2394	18
87	STL	LAX	1589	24
88		DEN	796	14

Part 1 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
89	STL	DAL	547	14
90		SEA	1705	10
91	LAX	SEA	959	26
92	ATL	MDW	587	40
93		NEW	424	29
94		STL	467	22
95		DAL	721	22
96		HOU	701	17
97	LAS	SFO	417	32
98	TPA	ATL	416	30
99		MDW	1003	17
100	BAL	MDW	606	23
101		ATL	577	14
102		MIA	955	12
103		LAX	2320	14
104	MKC	DEN	558	25
105		LAX	1356	16
106		DAL	451	12
107	BUF	MDW	454	21

Part 1 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
108	ABQ	LAS	484	10
109		LAX	664	14
110	SLC	LAX	590	16
111		LAS	508	16
112		SFO	600	16
113	MEM	DAL	420	22
114		HOU	478	12
115	NEW	DAL	443	23
116	TUS	LAX	450	14
117	CLT	MDW	584	10
118	MSP	DEN	700	14
119		LAX	1530	10
120		MKC	413	12
121	OMA	DEN	488	15
122	OGD	SAC	535	12
123	HAR	SAN	910	10

Part 1 - (Concluded)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
124	ICT	DEN	437	13
125	ROC	MDW	520	14

Total exchange of flights per day
between community pairs of Part 1 = 2,906

Part 2. Pairs employing essentially altitudes
between 18,000 ft. and 28,000 ft.

(Exchange of 10 or more flights/day)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
1	JFK	PIT	317	53
2		DCA	205	46
3		BUF	292	31
4		ROC	250	27
5		SYR	194	11
6	MDW	MSP	355	84
7		STL	262	63
8		DSM	309	19
9		CID	199	11
10	DET	MDW	238	80
11		MKE	252	17
12	HOU	DAL	225	53
13		NEW	316	44
14		SAT	189	14
15	MIA	TPA	205	54
16	CLE	MDW	308	50
17	BOS	PHL	271	31
18		JFK	188	19

Part 2 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
19	BOS	BAL	360	17
20		SYR	264	10
21		DCA	398	56
22	DCA	CLE	306	21
23		DET	396	20
24		PIT	192	20
25		DAY	380	16
26		CMH	329	10
27	DEN	SLC	371	23
28		ABQ	334	16
29	DAL	SAT	252	24
30		OKC	190	12
31		LBB	292	23
32	PHX	LAX	357	39
33		SAN	299	28
34		ONT	323	20
35	PIT	DAY	227	13
36		DET	205	13
37		CVG	257	12

Part 2 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
38	PHL	PIT	259	32
39		CLE	360	21
40	STL	MKC	238	35
41		TUL	361	14
42	LAX	SAC	361	34
43		PDX	357	18
44		FAT	204	17
45		SJC	307	70
46		SFO	337	237
47	ATL	SDF	319	21
48		MEM	337	18
49		BNA	214	17
50	CVG	MDW	252	29
51		STL	309	12
52	MKE	MSP	299	28
53	DAY	MDW	232	27
54	IND	STL	262	25
55	GEG	SEA	229	24
56		PDX	310	14

Part 2 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
57	MKC	ICT	177	13
58	BUF	DET	216	17
59		PIT	178	11
60	ABQ	PHX	330	20
61	SLC	BOI	296	19
62	RNO	SFO	185	20
63	CMH	MDW	252	19
64	MEM	STL	240	19
65		MKC	329	12
66		NEW	346	10
67	SDF	MDW	269	19
68	CAE	ATL	194	18
69	BNA	STL	254	17
70		MEM	197	12
71	CLT	ATL	227	14

Part 2 - (Continued)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
72	BHM	MOB	208	11
73		JAN	210	13
74	MSP	OMA	290	13
75		FAR	214	12
76		FSD	198	12
77	LTT	DAL	293	12
78	ELP	TUS	265	10
79		PHX	346	10
80	ALB	BUF	261	11
81	ABE	PIT	239	11
82	FAT	SFO	161	11
83		SJC	138	10
84	JAN	MEM	196	11
85	SAV	ATL	224	10
86	LAS	LAX	234	93
87		ONT	196	14

Part 2 - (Concluded)

Serial Number	Reference Community Designation	Designation	Paired Community Distance (App. Statute Miles)	Flight Exchange/Day
88	ONT	SFO	352	28
89	BOI	PDX	342	12
90	PHF	RIC	68	12
91	SYR	DCA	300	14
92	SAV	ATL	215	12
93	SAT	CVN	375	14
94	PVD	DCA	356	12
95	JAK	ATL	285	30
96	ICT	WDG	94	25
97	YUM	FAT	400	21
98	HFD	DCA	304	10

Total exchange of flights per day
between community pairs of Part 2 = 2,477

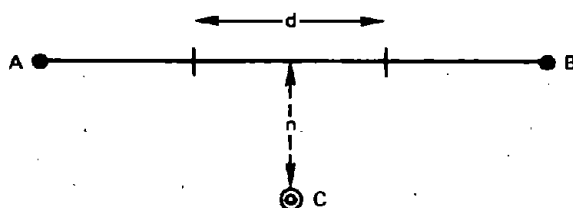
Total exchange of flights per day
between community pairs of Parts 1 and 2 = 5,383

Appendix I

A SIMPLE MODEL TO ESTIMATE THE FRACTION OF A ROUTE FLYABLE WITH ROUTE WIDTH W

1. Length of a Route Section Having Route Width W As a Function of the Normal Distance of a Supporting VORTAC.

Consider a route AB and a supporting VORTAC C as shown in Figure I-1. Let the normal distance of the VORTAC from the route be n nmi. Now referring to Figure 1, Appendix D, page 13 of the FAA Handbook 7110.18, "Air Traffic Control Service for Area Navigation Equipped Aircraft...", dated 27 February 1970, we have a relationship between the normal distance of a VORTAC (referred to as distance to point of tangency) and the length d of a route section having a width of 8 nmi and 4 nmi. Some typical values according to the Handbook are repeated in Table I-1 for convenience. We will use Table I-1 to develop a simplified expression giving the fraction of a route flyable with route width W when the supporting VORTAC grid is of triangular pattern with side length S .



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FIGURE I-1 A VORTAC SUPPORTING A ROUTE SECTION

Table I-1

ROUTE WIDTH SUMMARY

Distance of Route Section n(nmi)	Route Length d(nmi)	
	4 nmi	8 nmi
0	50	100
25	44	96
50	16	88
75	--	70
100	--	20

2. Fractions of a Route Having Width W as a Function of the Side Length S of a Triangular Grid.

Consider Figure I-2 where a triangular grid with side length S is shown along with routes OR, OQ, OP, and OT making angles of 0, 10, 20, and 30° with one of the reference rows of the triangular grid. Typically, route sections having a certain route width (e.g., 4 nmi) will consist of small patches along the route, as shown by the shading in Figure I-2. The length and distance between consecutive patches will be a function of the side length S and the angle between the route and a reference row of the VORTAC grid. The sum of the lengths of patches of a certain route width will be maximum when the route is laid over one of the rows of the VORTACs, e.g., along the route OR. Using Table I-1 as a reference, patches of 4 nmi and 8 nmi were generated along the routes OR, OQ, ... etc. making angles 10°, 20°, ... with S = 200 and S = 150 nmi and assuming a route length of 1,000 nmi. The length of patches of 4 nmi and 8 nmi route widths were summed along each route and divided by the total route length, i.e., 1,000 nmi, so that along each route the totals of the sections having a width of 4 nmi and 8 nmi

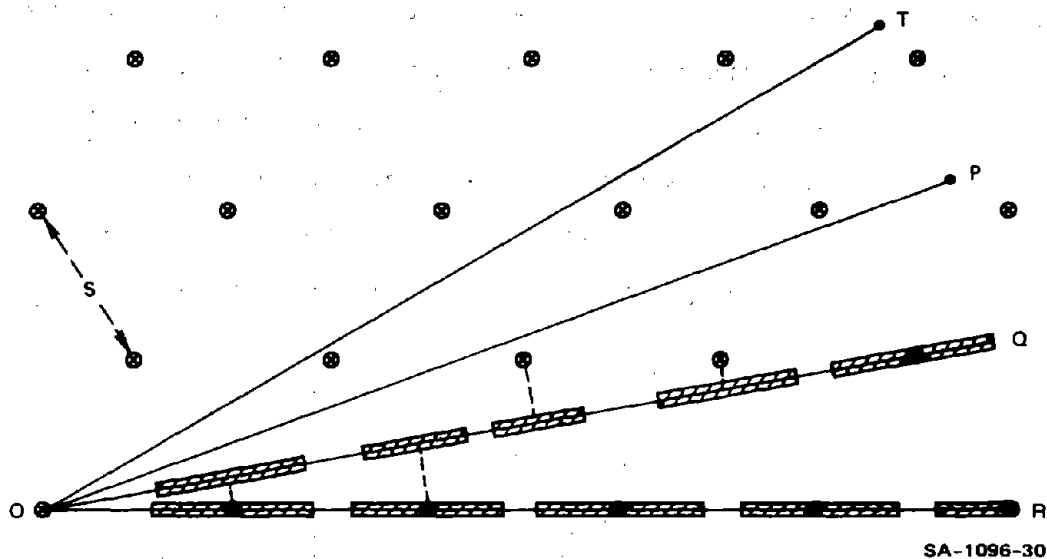


FIGURE I-2 ROUTE SECTIONS WITH CERTAIN ROUTE WIDTHS

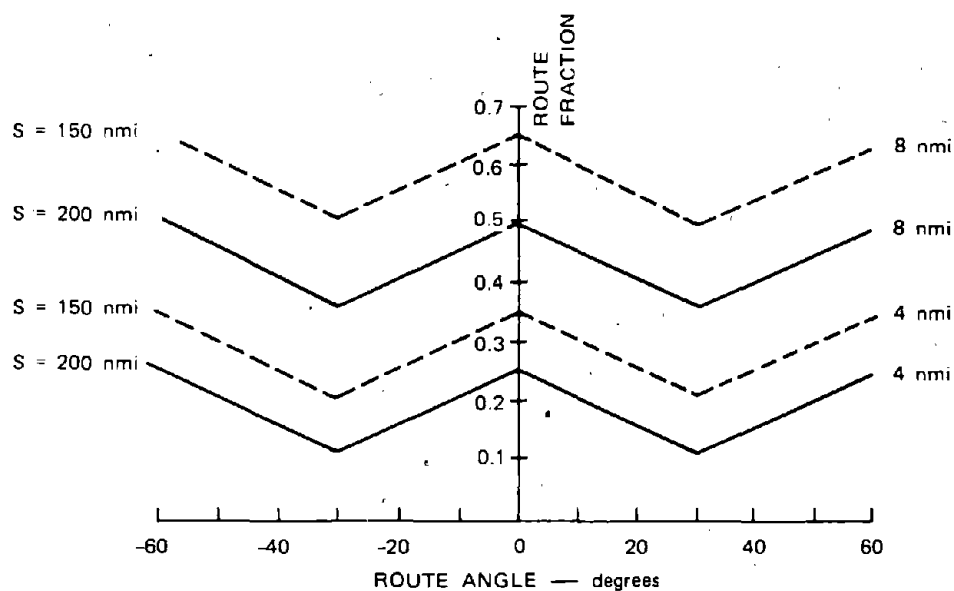
can be expressed as fractions of the route length. Some typical values are shown in Table I-2.

Table I-2

FRACTIONS OF A ROUTE WITH WIDTHS OF 4 NMI AND 8 NMI

Route Angle with Reference Row	S = 200 nmi		S = 150 nmi	
	4 nmi	8 nmi	4 nmi	8 nmi
0	.25	.50	.35	.65
±10	.20	.44	.30	.62
±20	.18	.42	.28	.58
±30	.115	.36	.20	.52
±40	.18	.42	.28	.58
±50	.20	.44	.30	.62
±60	.25	.50	.35	.65

The relationships in Table I-2 are depicted graphically in Figure I-3. Note that the fractions for routes making 10° and 50° angle with a reference row are identical, since a route that is inclined at 50° to one row is actually inclined at 10° to another row of the grid. Similarly, 20° inclination is equivalent to a 40° inclination and so on. This is because of the repetitive nature of a triangular VORTAC grid (every 60°).



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FIGURE I-3 ROUTE WIDTH SUMMARY FOR ROUTE SECTIONS (S = 150 AND S = 200 nmi)

Without any significant error, a linear fit can be made for the data points as shown in Figure I-3. If we now assume that for other values of S and W, similar fits can be established using linear relationships, i.e., if side length S is increased by a factor k_1 , the fractions for a certain route width will decrease by a factor k_1 --note that larger values of S mean a smaller number of patches of a certain route width for a fixed route length--and if W is chosen higher or lower a corresponding increase or reduction in the fractions would occur, then

using the S=200, W = 4 nmi set of data, a relationship giving the fraction of a route having a width W, side length S, and route angle θ with one of the reference rows of the grid can be expressed analytically as follows:

$$F_V = \frac{200}{S} \left[0.25 + \frac{.25(W - 4)}{4} - \frac{.125 \theta}{30} \right] \quad \begin{array}{l} \text{for } 0^\circ \leq |\theta| \leq 30^\circ \\ \text{and } 60^\circ \leq |\theta| \leq 90^\circ \end{array}$$

$$= \frac{200}{S} \left[0.25 + \frac{.25(W - 4)}{4} - \frac{.125(60 - \theta)}{30} \right] \quad \text{for } 30^\circ \leq |\theta| \leq 60^\circ$$

where F_V = fraction of the route having route width W.

The above relationship is of course not exact, but seems to be adequately correct for a gross evaluation of matching between a route network and a given triangular VORTAC grid.

BIBLIOGRAPHY

The following technical notes, technical memoranda, working papers, and file notes were prepared by the NAFEC and SRI team in connection with the project for the purpose of informal discussions and tentative documentation of seemingly useful results. Portions of these documents have been used in preparing the present report. Many of these documents are self-contained and may be useful in further work on air traffic problems.

Technical Notes Prepared by NAFEC

B. Flex and F. B. Woodson, "Derivation for Equation of Great Circle," Technical Note No. 3 (August 1970).

F. B. Woodson and S. J. O'Kane, "Derivation of Math Used in Program To Find the Normal Distance between a Given Fix and a Given Great Circle, and To Find the Latitude and Longitude of the Intercept," Technical Note No. 4 (August 1970).

S. J. O'Kane, "Derivation of Math Used in Program To Solve for the Point of Equal Cross Course Error," Technical Note No. 5 (August 1970).

R. W. Soper, "Calculation of Added Travel Distance to an Airport Offset from a Common Air Route," Technical Note No. 6 (February 1971).

P. Letzter, "Initializing Program Library for High Altitude Area Navigation Airway Development," Technical Note No. 7 (April 1971).

H. Arch, C. M. Russell, and R. Soper, "Methods for Initializing HIRNAV Airways," Technical Note No. 8 (May 1971).

R. W. Soper, "Calculation of Turnoff Angle--An Airport Offset from a Common Air Route," Technical Note No. 9 (June 1971).

F. B. Woodson and R. W. Soper, "Mathematical Derivation of Earth's Radius and Nautical Miles per Degree Given Latitude," Technical Note No. 10 (June 1971).

Technical Memorandums Prepared by SRI

A. J. Korsak, "Derivation of Differential Equations for Geodesics on a Sphere and an Oblate Spheroid," Technical Memorandum No. 1 (March 1971).

A. J. Korsak, "Derivation of a Formula for Latitude and Longitude as Functions of Arc-Length along a Great Circle," Technical Memorandum No. 2 (March 1971).

A. J. Korsak, "A Proposed Approach for Assigning Altitudes to Airways so as to Minimize Intersections at Same Altitude," Technical Memorandum No. 3 (March 1971).

M. W. Siddiquee, "Conflict Prediction Model Based on Radar Separation Rules," Technical Memorandum No. 4 (June 1971).

M. W. Siddiquee, "A Modification in Conflict Prediction Model Based on Radar Separation Rules," Supplement to Technical Memorandum No. 4 (September 1971).

M. W. Siddiquee, "Verification of Conflict Prediction Model," Technical Memorandum No. 5 (July 1971).

R. Dressler (SRI) and R. Soper (NAFEC), "Calculating the Intersection Point of Two Great-Circle Routes," Technical Memorandum No. 6 (July 1971).

M. W. Siddiquee, "Conflict Duration and Capacity at Intersecting Routes," Technical Memorandum No. 7 (September 1971).

Working Papers Prepared by NAFEC and SRI

A. J. Korsak, "A Proposed Approach for Assigning Altitudes to Airways so as to Minimize Intersections at Same Altitude," Working Paper No. 1 (March 1971).

R. M. Dressler, "Task 3: Problem Definition and Ground Rules," Working Paper No. 2 (March 1971).

M. W. Siddiquee, "Task 3: Criterion for Testing the Quality of a Route Network," Working Paper No. 3 (March 1971).

R. M. Dressler, "Preliminary Considerations for the Traffic Demand Model," Working Paper No. 4 (April 1971).

A. J. Korsak, "Elaboration on the Altitude Assignment Problem," Working Paper No. 5 (March 1971).

R. M. Dressler, "Task 4: Initial Considerations," Working Paper No. 6 (April 1971).

F. Woodson, C. Russell, H. Arch, and R. Soper and R. Dressler and W. Siddiquee, "Problem Definition 'Ground Rules' for Tasks 3 and 4," Working Paper No. 8 (May 1971).

W. Siddiquee and R. Dressler and C. Russell, F. Woodson, H. Arch, and R. Soper, "Tentative Criteria and Ground Rules for Grouping Threads into Ropes," Working Paper No. 9 (July 1971).

File Notes Prepared by SRI

W. Siddiquee, "Some Aspects of Grouping Threads into Ropes," File Note No. 5 (April 1971).

W. Siddiquee, "Expected Number of Conflicts at a Merge Point of Two Air Routes," File Note No. 7 (May 1971).

W. Siddiquee, "Network Effectiveness Criteria Revisited," File Note No. 8 (May 1971).

W. Siddiquee, "On Altitude Assignment Policy," File Note No. 10 (June 1971).

W. Siddiquee, "Exploring a New Concept in Altitude Assignment Problems," File Note No. 11 (July 1971).

W. Siddiquee, "On Bending a Route to Remove Intersections," File Note No. 13 (July 1971).

A. Korsak, "Characterization of Intersecting and Non-Intersecting Airways in Terms of their Terminal Coordinates," File Note No. 14 (July 1971).

W. Siddiquee, "Grouping of Threads on the Basis of a Minimum Departing Angle between Merged and Primary Threads," File Note No. 16 (July 1971).

W. Siddiquee, "On Assigning an Altitude Other than the Desired One," File Note No. 17 (August 1971).

W. Siddiquee, "Air Route Capacity Models," File Note No. 19 (September 1971).

W. Siddiquee, "Air Route Capacity Models," File Note No. 20 (September 1971).

W. Siddiquee, "Air Route Capacity Models," File Note No. 21 (September 1971).

